Coupling Analysis for the Thermo-Acoustic-Vibration Response of a Thin-Walled Box with Acoustic Excitations

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Abstract

Monitor and control unit is essential to almost all of the modern aircrafts or spacecrafts, which always places in a relatively closed space and requires a compatible mechanical environment to ensure its normal function. The acoustic, vibration and thermal loads developed in the operation of the aircraft determine a complex environment for its monitor and control unit. In particular, there should be couple effect among the vibration and acoustic responses. This article presents numerical simulation studies on a thermo-acoustic-vibration response of a typical thin-walled box, to explore the multi-physics environment of the monitor and control unit. The thermo-structural equations were proposed and solved numerically, following that the vibro-acoustic analysis was implemented. The coupled algorithm was developed to simulate the structure response and sound distribution, and predict the sound loss during transmitting through the box. The outcome provides the multi-physics environment prediction method for designing and optimizing the monitor and control unit or similar applications.

Keywords: Thermal, Vibration, Acoustic, Multi-physics environment

List of Symbols

$\boldsymbol{U}$, $\dot{\boldsymbol{U}}$ and $\ddot{\boldsymbol{U}}$	Displacement, velocity and acceleration vectors of the solid
$\boldsymbol{M}$	Structural mass matrix
$\boldsymbol{C}$	Structural damping matrix
$\boldsymbol{K}$ and $\boldsymbol{K_\sigma}$	Temperature-dependent stiffness matrix and its increment induced by thermal stress
$\boldsymbol{F^t}$, $\boldsymbol{F_s}$	Load force vector and its harmonic amplitude; $\boldsymbol{F^t(t) = F_s e^{j\omega t}}$
$\boldsymbol{F_a}$	Sound source vector
$\boldsymbol{p}$, p	Acoustic pressure vector and its scalar
q	Sound source
$\boldsymbol{R_s}$	The coupled matrix describing acoustic pressure acting upon the structure
k	Wave number, $k = \omega/c = 2\pi f/c$
ρ_0	Density of the fluid
$\boldsymbol{G}$, $\boldsymbol{H}$	Boundary integral influence matrix
$\boldsymbol{u}_f$	Displacement of fluid at the fluid-structure interface
SPL	Sound pressure level, defined by $SPL = 20\log(p_{\text{rms}}/p_{\text{ref}})$, $p_{\text{ref}} = 2.0 \times 10^{-5}$ Pa
P	Acoustic power
TL	Transmission loss, through the structure

1 Introduction

Acoustic-vibration is an important coupled mechanical environment for modern aircrafts or spacecrafts, especially for the monitor and control unit, which comprises precision instruments and behaves sensitive to strong vibration and noise. In general, vibro-acoustic responses experiments should be performed in the design and development stages[1]. For decades, numerical simulation and prediction techniques have been developed increasingly to help the design and optimization of the

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mechanical environment of spacecrafts devices. As to vibro-acoustic analysis, coupled Finite Element Method (FEM) and Boundary Element Method (BEM) has been widely used, in order to reduce the computation cost.

As known, when the environmental temperature changes, the structure property will change, in particular the elastic modulus of the materials, which affects the material properties and vibro-acoustic behavior of the structure[2]. Not much work has been done in the thermo-vibro-acoustic analysis for multiple physics. Jeyaraj *et al.*[2] studied the vibration and acoustic response of a plate in a thermal environment subjected to a harmonic load, in which the thermal stress effect is included in a so called geometric stiffness matrix without considering the softening of the materials due to temperature elevation. Geng and Li[3] studied the vibration and acoustic radiation characters of an isotropic rectangular thin plate under thermal environments, in which the thermal structure stress is assumed to be the only contribution made by the thermal environment.

In the preceding work[4], the thermo-vibro-acoustic coupling model has been proposed. In this study, the thermo-vibro-acoustic analysis was implemented to investigate the multi-physics environment of a typical thin-walled box model with varied acoustic sources, to simulate the acoustic and vibration responses of the monitoring and control unit of a spacecraft. The paper is organized as below. Firstly, the coupled mechanical model and numerical methods are descripted. Secondly, two typical cases are presented, 1) a plane wave exerted outside the box, 2) a simple point acoustic source, monopole and dipole, in the box. Finally, the conclusions according to the results are drawn.

2 Methods

The coupled algorithm comprises two folds. The thermo-structural equations were proposed and solved numerically with Finite Element Method (FEM), and the vibro-acoustic analysis was made by using Boundary Element Method (BEM). In detail, the temperature and thermal stress are calculated by FEM for every thermal equilibrium state with considering the thermal softening of the materials. The vibration and acoustic response are computed by BEM for typical thermal equilibrium state.

2.1 Thermo-vibro-acoustic equation and numerical methods

The equation of motion for a pre-stressed structure with the sound pressure load is as follows[2],

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{U}}\} + [\boldsymbol{C}]\{\dot{\boldsymbol{U}}\} + [\boldsymbol{K} + \boldsymbol{K}_\sigma]\{\boldsymbol{U}\} = \{\boldsymbol{F}^t\} + \{\boldsymbol{R}_s \boldsymbol{p}\} \tag{1}$$

where $[\boldsymbol{M}]$ is the inertial matrix, $[\boldsymbol{U}]$ is the displacement vector, $[\boldsymbol{C}]$ is the structural damping matrix, $[\boldsymbol{K}]$ is the temperature-dependent stiffness coefficient matrix, $[\boldsymbol{K}_\sigma]$ is the increment stiffness coefficient matrix induced by thermal stress, $[\boldsymbol{F}^t]$ is the applied load vector (supposed time harmonic), $\mathbf{p}$ is the acoustic pressure, and $\boldsymbol{R}_s$ is the coupled matrix describing acoustic pressure acting upon the structure.

For harmonic solution when $\boldsymbol{F}^t(t) = \boldsymbol{F}_s e^{j\omega t}$, assuming $\boldsymbol{U}(t) = \boldsymbol{U}e^{j\omega t}$, the (1) turns into,

$$([\boldsymbol{K} + \boldsymbol{K}_\sigma] + j\omega \boldsymbol{C} - \omega^2[\boldsymbol{M}])(\{\boldsymbol{U}\}) + \boldsymbol{L}_c\{\boldsymbol{p}\} = \{\boldsymbol{F}_s\} \tag{2}$$

where $\boldsymbol{L}_c$ is the vibro-acoustic coupled matrix, $\boldsymbol{F}_s$ is the harmonic amplitude of the force vector.

The basic governing equation of sound wave is Helmholtz equation (in frequency domain), assuming the sound wave is harmonic,

$$\nabla^2 \boldsymbol{p} - \boldsymbol{k}^2 \boldsymbol{p} = -j\rho_0\omega \boldsymbol{q} \tag{3}$$

where p is acoustic pressure, $k = \omega/c = 2\pi f/c$ is wave number, ρ_0 is the density of the fluid, and q is the sound source. The boundary element method for acoustic problem is written as[5],

$$\rho_0\omega^2[\boldsymbol{G}]\boldsymbol{u}_f + [\boldsymbol{H}]_p = \boldsymbol{F}_a \tag{4}$$

where $\boldsymbol{G}$ and $\boldsymbol{H}$ are boundary integral influence matrices, $\boldsymbol{F}_a$ is the sound source vector, $\boldsymbol{p}$ and $\boldsymbol{u}_f$ are the acoustic pressure and displacement of the boundary element nodes of the fluid. The coupled FEM/BEM is implemented to solve the structural-acoustic problem, using ANSYS and SYSNOISE respectively. First the prestressed harmonic analysis is performed in ANSYS, the result file is imported into SYSNOISE to find the acoustic-structural response, where the indirect harmonic BEM is used.

2.2 The model

The model was set up as a thin-walled box, as Fig. 1 shows, with geometry $0.5 \times 0.3 \times 0.2\,\mathrm{m}^3$, with thickness 2.5 mm. The material is specified as steel, with Young's modulus $E = 200\,\mathrm{GPa}$, Poisson's ratio 0.3, mass density 7800 kg/m3, thermal expansion coefficient 1.0×105 under ambient temperature condition. The acoustic medium is modeled as air, with density

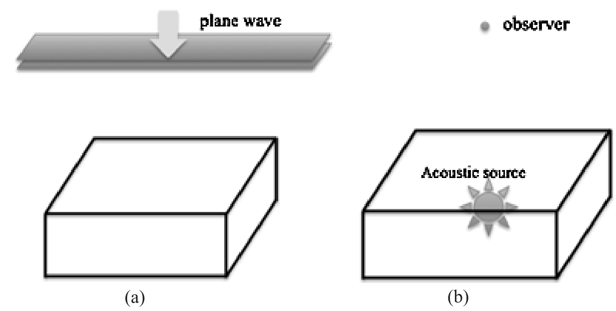


Fig. 1 The sketch of the model

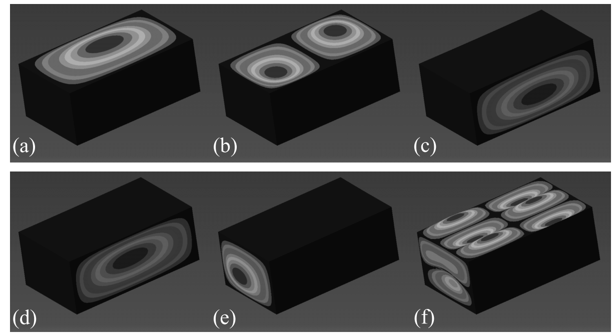


Fig. 2 The mode shapes, Mode 1, 2, 3, 4, 5 and 25 (a–f), of the box under different ambient temperature

Table 1 Natural frequencies of the box under different ambient temperature

Natural Frequencies (Hz)	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 25
Cool	179.3	299.0	391.5	394.8	435.9	1526.1
Hot	150.0	250.1	327.5	330.3	364.7	1276.8

1.21 kg/m^3 and sound speed 340 m/s. The fluid material properties are supposed to be temperature-independent in this study. Without loss of generality, one normally incident harmonic plane sound wave, with sound pressure level 140 dB, is imposed upon the top of the box, to represent an acoustic source outside, as shown in Fig. 1(a), and a simple acoustic source, namely a monopole and a dipole is imposed in the center of the box respectively, like Fig. 1(b). In this study, there is no force excitation, in order to focus only on the acoustic environment.

Two cases, denoted by 'cool' and 'hot', where two values of Young's modulus determined by the material temperature are specified, that is, in the hot case the material is supposed to has 70% Young's modulus than that of the cool one, i.e., 140 GPa in magnitude.

The volume element mesh of FEM model is matched with the shell element mesh of BEM, where the shell element size is set as 10 mm, and the thickness of volume element is equal to the thickness of the box, 2.5 mm. Such mesh size can resolve the acoustic analysis up to 5666 Hz.

3 Results and Discussion

3.1 The natural modes

The four corner points in the bottom wall plate of the box are fixed in computation. The first 25 modes were analysis with FEM. Table 1 shows the first 5 and the 25th natural frequencies of the box in cool and hot conditions. It is clear that the natural frequencies of the box reduce with increase in temperature. The first 2 mode shapes are shown in Fig. 2.

3.2 Vibro-acoustic response characteristics

The vibro-acoustic response may be represented by the acoustic field response at the center point of the box. Figure 3 is the sound pressure level at the center point (SPL_{center}), and the transmission loss (TL) here is defined as,

$$TL_{(1)} = SPL_{\text{in}} - SPL_{\text{center}} \tag{5}$$

where SPL_{in} is the sound pressure level of the incident sound wave, which is 140 dB.

The result shows that, around every natural frequency of the structure, the sound transmission would appear a corresponding peak. The transmission loss of the 'hot' box is slightly less than that of the 'cool' one. However, in higher frequency bands, above about 1500 Hz, such difference would disappear.

The acoustic response in the middle-'Z' section is given, as Fig. 5 shows. The sound wave transmits into the box; the acoustic field in the box appears well-distributed in the box, in the region near the wall, the sound pressure level decreases soon.

Furthermore, the vibration response can be found, shown in Fig. 6. It is seen that the displacement response shape is similar with the natural mode shape at the same frequency, by comparing Fig. 6 and Fig. 2. Thus, the resonance effect was numerically simulated.

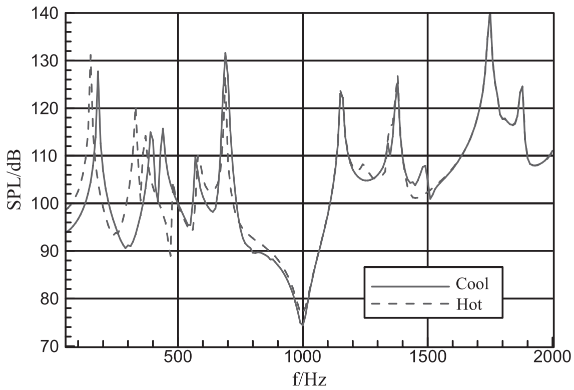


Fig. 3 The acoustic response, the sound pressure level ($p_{\mathrm{ref}} = 2.0 \times 10^{-5}$ Pa) at the center point of the box

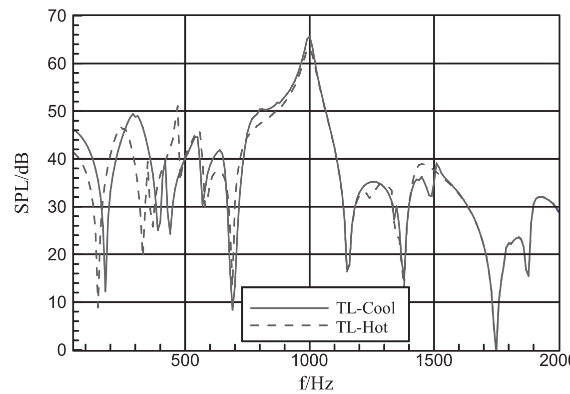


Fig. 4 The transmission loss, $TL_{(1)}$ of the box

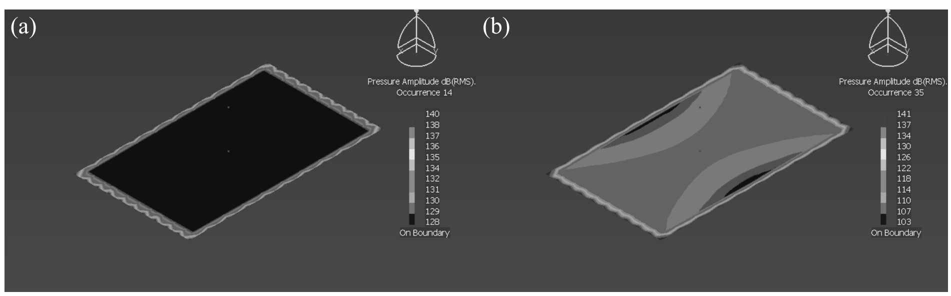


Fig. 5 The sound pressure level contour in the middle-‘Z’ section, the cool case at 180 Hz (a) and 430 Hz (b)

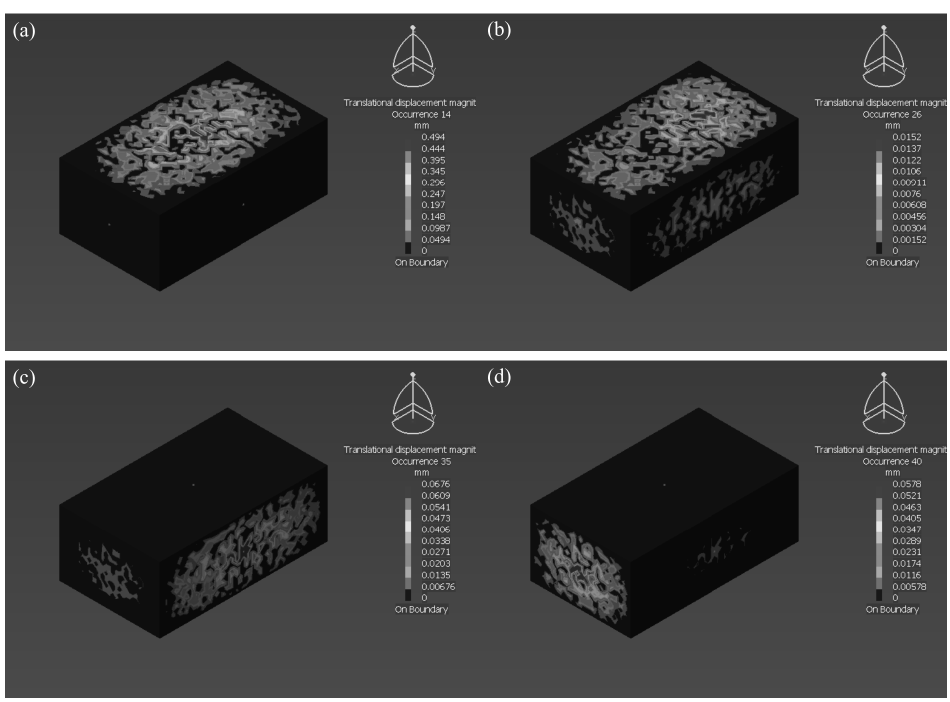


Fig. 6 The displacement response magnitude contour, on the inner surface of the ‘cool’ box, at 180 Hz (a), 300 Hz (b), 390 Hz (c) and 440 Hz (d)

The sound power on the middle-'Z' section, P_{mid}, is obtained, thus the transmission loss (TL) can be computed in another expression,

$$TL_{(2)} = 10 \log(P_{\text{in}}/P_{\text{mid}}), \quad \text{where} \quad P_{\text{in}} = \frac{p_{\text{rms}}^2}{\rho_0 c} S \tag{6}$$

and S is the area of the top face of the box, and P_{in} is the sound power of the incident sound wave. Figure 7 gives the sound power level and the transmission loss, $TL_{(2)}$, according to formula (6). The result shows that, the difference caused by temperature is not so much. However, the qualitative property of the transmission loss is the same, comparing with the result of $TL_{(1)}$, that is, the transmission loss of the 'hot' box is a little less than that of the 'cool' one and the valley frequencies is accord with the natural frequencies of the structure. Besides, in higher frequency bands, above 700 Hz around, the difference tends to disappear.

For the center point sound pressure level, as formula (5) defined, the transmission loss ($TL_{(1)}$) is much greater than that defined by (6) ($TL_{(2)}$), because the center point is a sensitive point, where the vibro-acoustic coupling is strong, than the points near the inner wall of the box and $TL_{(2)}$ reflects the total vibro-acoustic coupling effect.

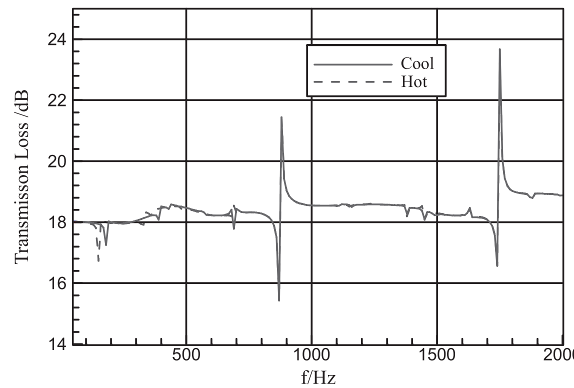
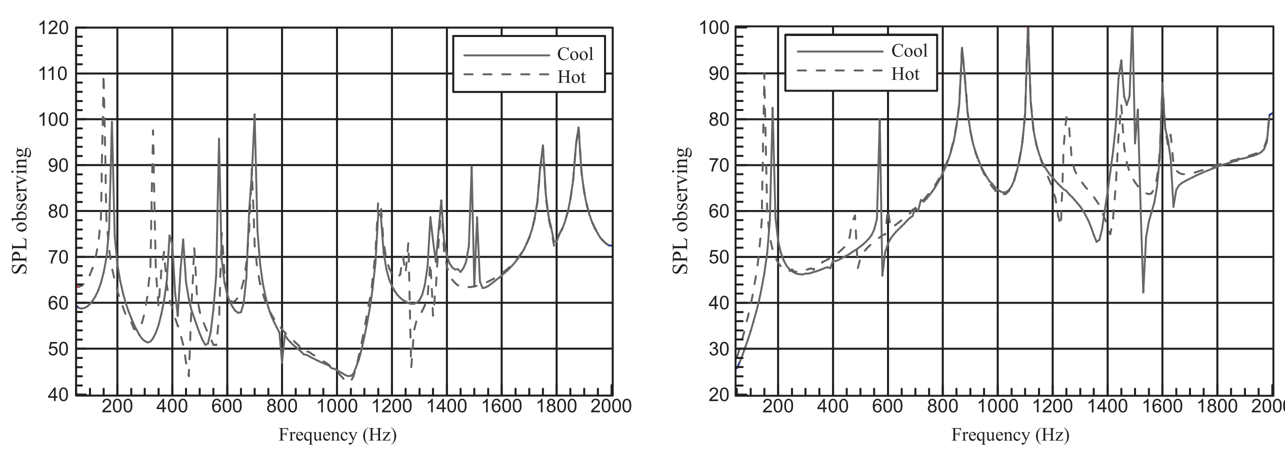


Fig. 7 The transmission loss, $TL_{(2)}$ (defined by equation 6) of the box

3.3 Vibro-acoustic response to varied acoustic sources

Two kinds of acoustic sources, namely monopole and dipole (on z-direction), were placed at the center of the box, their vibro-acoustic responses were investigated. For monopole, the acoustic source amplitude is $A = 1.0$ N/m, where its incident pressure is $p(r) = A \exp(-jkr)/r$; for dipole, the acoustic dipole strength is $D = 1$ N, with the definition $\boldsymbol{p(x) = D_i \frac{\partial G(x|y)}{\partial y_i}}$, where G is the free-field Green's function. Figure 8 shows the acoustic response at the observing point out of the box. At the natural frequencies, peaks appear apparently, particularly at the first several modes, which indicates that the natural structural vibration has remarkable positive contributions to the structure-borne propagation of sound.

Figure 9 and Fig. 10 give the structural response on the box. It is shown that, the natural modes of vibration mainly determine the vibration excited by the sound. However, varied sound sources gain different vibration, especially when the acoustic sources have its distinguishing property on directivity. Comparing Fig. 9(c) with Fig. 10(c), and Fig. 9(d) with Fig. 10(d), the principle difference is that the vibration excited by the monopole induces larger vibration in z-direction and smaller in x- and y-direction, relatively.



(a) With a monopole acoustic source

(b) With a dipole acoustic source

Fig. 8 The acoustic response, the sound pressure level ($p_{\text{ref}} = 2.0 \times 10^{-5}$ Pa) at the outer observing point, $z = 0.5$ m, with the origin point set on the bottom of the box

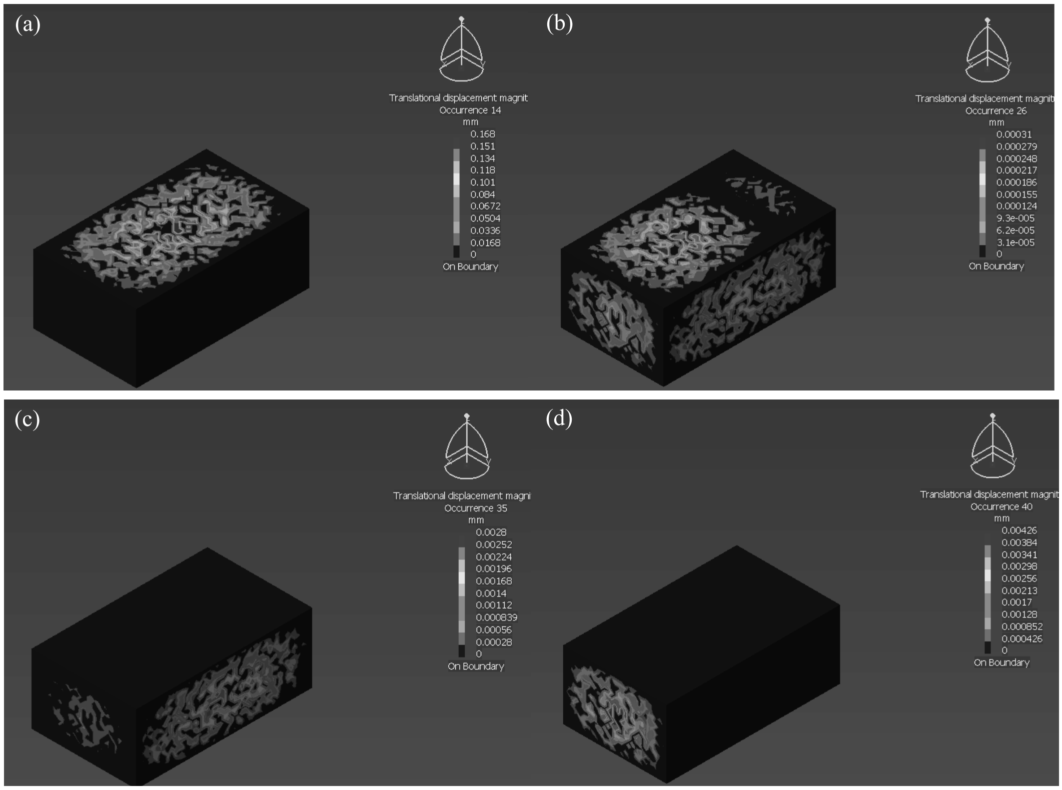


Fig. 9 The displacement response magnitude contour, with a monopole acoustic source imposed, on the surface of the 'cool' box, at 180 Hz (a), 300 Hz (b), 390 Hz (c) and 440 Hz (d)

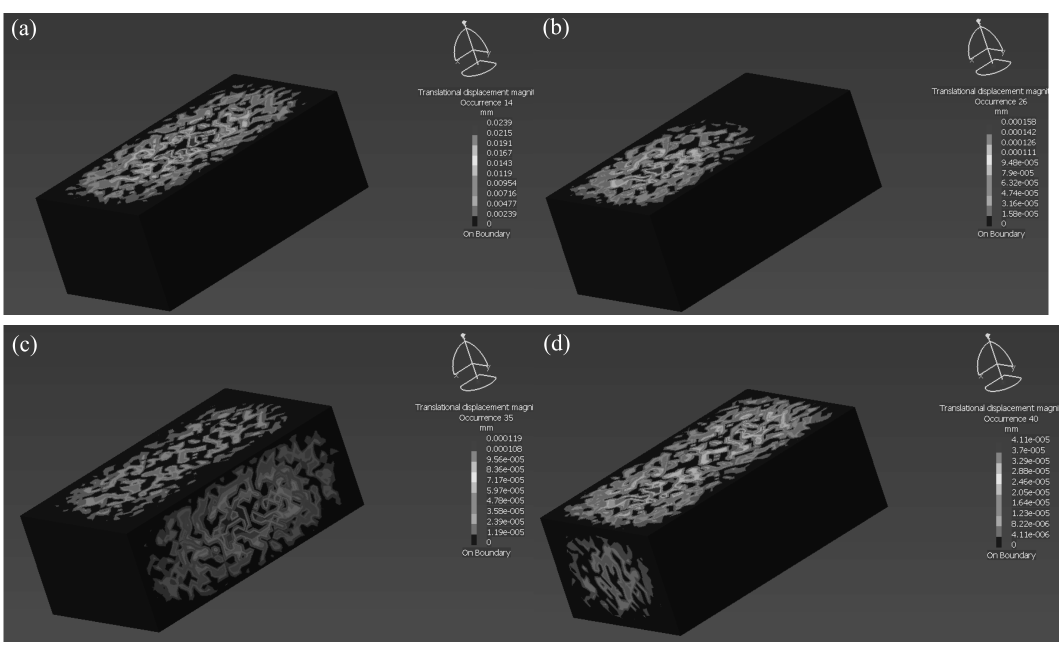


Fig. 10 The displacement response magnitude contour, with a dipole acoustic source imposed, on the surface of the 'cool' box, at 180 Hz (a), 300 Hz (b), 390 Hz (c) and 440 Hz (d)

4 Conclusions

The paper gives a thermo-vibro-acoustic analysis of a thin-walled box, in order to investigate the multi-physics characteristics. The coupled FEM/BEM is implemented to analyze the structural and acoustic response. The effect of thermal environment is studied to consider both thermal stress and thermal softening of the materials, with a plane acoustic wave exerted from the ambient, and with one simple acoustic source in the box. The acoustic and vibration responses are given, to observe the vibro-acoustic characteristics. Results show that corresponding to the frequencies of the natural vibration modes, the acoustic response at the observing point exhibits peaks in the spectrum, which suggests that the natural vibrations increase the structural-borne propagation. The natural vibration dominates the vibration response, and sound stimulation induces small variations on local structures. It is revealed that the acoustic responses around the low frequency would be obviously changed by the temperature elevation of the material, while such difference would disappear in the region of high frequency, which correspondingly affects the acoustic and vibration responses.

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