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环形液池热毛细对流的线性稳定性研究*

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摘 要 对外壁加热的环形液池热毛细对流进行了线性稳定性分析. 采用 Chebyshev 配点法对 $Pr = 6.8$、内外径之比为 0.5、深宽比 A 范围为 0.25~1.4 的数值结果进行分析, 发现流动的临界状态均为振荡形式, 并且随着 A 的增大, 临界雷诺数减小, 相应的临界波数与振荡频率也呈减小趋势. 能量分析结果表明, 小扰动与基本流相互作用项较小, 表面张力在径向做功与周向做功对小扰动的动能变化起主导作用. 观察三者与液池深宽比的关系, 发现 $A = 0.8$ 时表面张力在径向做功项达到极小值, 周向做功项以及小扰动与基本流相互作用项达到极大值.

关键词 热毛细对流, 环形液池, 线性稳定性分析

中图分类号 V524

Linear Stability Analysis of Thermocapillary Convection in Annular Pools

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Abstract The linear stability of thermocapillary convection in annular pools is studied through the Chebyshev-collocation method. As $Pr = 6.8$, the ratio of outer radius and inner radius is 0.5, and the range of aspect ratio A is from 0.25 to 1.4, numerical results show that the critical mode of the flow are oscillating. The critical Marangoni number, critical wave number and oscillating frequency decreases as A increases. Energy analysis shows that surface tension in the radial and azimuthal directions plays a leading role in the variation of perturbation energy. And the interaction between perturbation flow and the basic flow is small compared to the two former terms. We find out that the work done by the surface tension in the radial direction reaches minimum were analyzed, while the work done by the surface tension in the azimuthal direction and the interaction between the perturbation flow and the basic flow reach maximum, as $A = 0.8$.

Key words Thermocapillary convection, Annular pools, Linear stability analysis

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0 引言

由于热毛细流动在晶体生长技术与微重力材料加工 [1] 方面的巨大实用价值, 对热毛细流动的研究一直受到广泛关注. 目前已经提出了一些热毛细对流模型, 例如液层、液桥和方腔等 [2-3]. 环形液池是其中被研究最多的简单模型之一, 环形液池中热毛细对流的诸多相关实验已经在空间与地面开展. Kamotani 等 [4-5] 进行了很多大 Pr 数 (Pr 为 22~32) 流体的微重力实验, 实验结果表明自由面变形在振荡机理中具有很大作用. Mukolobwiez 等 [6] 与 Garnier 和 Chiffaudel[7] 得到了正常重力下 0.65 cSt 硅油超临界失稳的实验结果, 并在浅液池中观察到热流体波. 目前已有 0.65 cSt 硅油环状热毛细对流的微重力实验 [8-10] 与数值研究 [11], 二者得出的结果相互定性满足但定量不满足. Li 等 [12] 对环形液池中的热毛细流动进行了三维数值模拟, 发现两种不同类型的振荡. 通过实验与数值模拟, 有研究对重力在热毛细流动中的影响进行了分析, 发现重力在浅液池中对流动起稳定作用 [8-9,13-14], 而 Rayleigh-Bernard 失稳发生在深液池中 [15]. Schwabe[16] 进行了浮力 - 热毛细对流实验, 发现引起振荡的临界温差与液池深度无关. 关于旋转效应对稳定性的影响, 浅液池中的数值模拟 [17-20] 结果表明, 在 Pr =6.7 时旋转使得流动不稳定, 而在 $Pr = 0.011$ 时则刚好相反. 尽管大多数关于环形液池热毛细对流的研究均是围绕外壁加热情况进行的, 但实际上内壁加热情况也受到关注 [4-6,19,21]. 内壁加热与外壁加热情况相比, 临界温差更小 [19]. 此外, 还有一些研究采用线性稳定性分析方法 [17,20,22-24], 主要集中在浅液池, 流体为 0.65 cSt 硅油和硅熔体.

本文采用线性稳定性分析方法, 对 $Pr = 6.8$ 下不同深宽比的环形液池热毛细对流进行研究, 进而通过能量分析研究流动失稳的机理.

1 控制方程

环形液池模型如图 1 所示. 典型无量纲参数例如深宽比、Marangoni 数、Prandtl 数、毛细 Reynolds 数和 Biot 数分别定义为

$$A = \frac{H}{R_o - R_i}, \quad Ma = \frac{|\sigma'_T|\Delta T(R_o - R_i)}{\rho_0 \nu \alpha},$$

$$Pr = \frac{\nu}{\alpha}, \quad Re = Ma/Pr, \quad Bi = \frac{hR_o}{k}.$$

其中, H 为高度, R_o 为外径, R_i 为内径, ρ_0 为液体密度, ν 为运动粘度, α 为热扩散率, k 为热导率, h 为传热系数, σ'_T 为表面张力温度系数. $\Delta T = T_o - T_i$ 为外壁与内壁间的温差.

为了推导环状流的无量纲化控制方程, 设定特征长度、时间与速度分别为 $R_o - R_i$, $(R_o - R_i)^2/\nu$ 与 $\nu/(R_o - R_i)$. 设内外径之比 $R_i/R_o = 0.5$, 则无量纲化控制方程如下:

$$\begin{aligned} &\nabla \cdot \boldsymbol{u}_0 = 0, \\ &\frac{\partial \boldsymbol{u}_0}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}_0 = -\nabla p + \nabla^2 \boldsymbol{u}_0, \\ &\frac{\partial T_0}{\partial t} + \boldsymbol{u}_0 \cdot \nabla T_0 = \frac{1}{Pr}\nabla^2 T_0. \end{aligned} \tag{1}$$

其中, 下标 0 表示基本流. 采用线性稳定性分析方法, 小幅振荡速度 $\boldsymbol{u} = (u, v, w)$, 压力 p 与温度 T 叠加到基本流上. 叠加扰动后的无量纲控制方程如下:

$$\begin{aligned} &\nabla \cdot \boldsymbol{u} = 0, \\ &\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}_0 + \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u} = -\nabla p + \nabla^2 \boldsymbol{u}, \\ &\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T_0 + \boldsymbol{u}_0 \cdot \nabla T = \frac{1}{Pr}\nabla^2 T. \end{aligned} \tag{2}$$

扰动量 (u, v, w, p, T) 可被展开为

$$\begin{aligned} (u, v, w, p, T) = &\sum_m \exp(\sigma t + \mathrm{i}m\theta) \cdot \\ &[\tilde{u}_m(r,z), im\tilde{v}_m(r,z), \tilde{w}_m(r,z), \\ &\tilde{p}_m(r,z), \tilde{T}_m(r,z)] + c.c. \end{aligned} \tag{3}$$

其中, $\sigma = \sigma_r + \mathrm{i}\sigma_i$, 这里 σ_r 和 σ_i 分别为小扰动的增

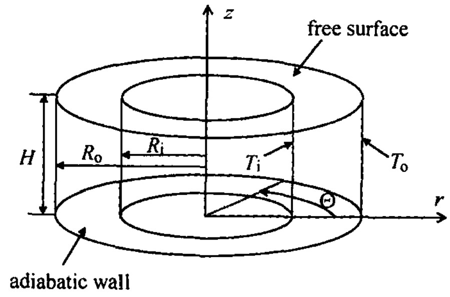

图 1 环形液池模型

Fig. 1 Annular pool model

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长速率和频率, $c.c.$ 表示复数共轭项, m 表示扰动波数. 基本流与扰动求解采用 Chebyshev 配点法.

2 数值结果

不同深宽比下的临界 Ma 数随波数的变化如图 2 所示, 从图 2 可以看出临界 Ma 数与临界波数一样, 随着深宽比的增大而减小. 与实验数据及三维数值模拟结果相比, 临界 Ma 数的线性稳定性分析结果明显偏小, 其原因显然与线性失稳后的非线性反馈机制有关.

不同深宽比下自由面扰动温度分布如图 3 所示. 由图 3 可以看出, 扰动多集中在热端, 且不同深宽比下扰动分布有很大不同.

3 能量分析

扰动能量的变化率可写为

$$\frac{\partial E_{\rm kin}}{\partial t} = -\frac{1}{2}\int (\boldsymbol{S}:\boldsymbol{S})\mathrm{d}^3r + \int \boldsymbol{u}\cdot\boldsymbol{S}\cdot\boldsymbol{n}\mathrm{d}^2r - \int \boldsymbol{u}\cdot[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}_0]\mathrm{d}^3r = -D_{\rm v} + M + I_{\rm v}, \tag{4}$$

其中, $\boldsymbol{S}=\nabla\boldsymbol{u}+\boldsymbol{u}\nabla$ 为应变率张量, $\boldsymbol{S}:\boldsymbol{S}$ 表示张量的双点积, $\boldsymbol{n}$ 为自由面的法向单位矢量, D_v 为粘性耗散项, $I_{\rm v}$ 为扰动流与基本流间的相互作用项. M 为表面张力在自由面上的做功, 可分解为 $M=M_r+M_\theta$, 这里 M_r 和 M_θ 分别表示表面张力在径向与周向的做功. 由于粘性耗散项 $D_{\rm v}$ 始终为正, 这里以其作为

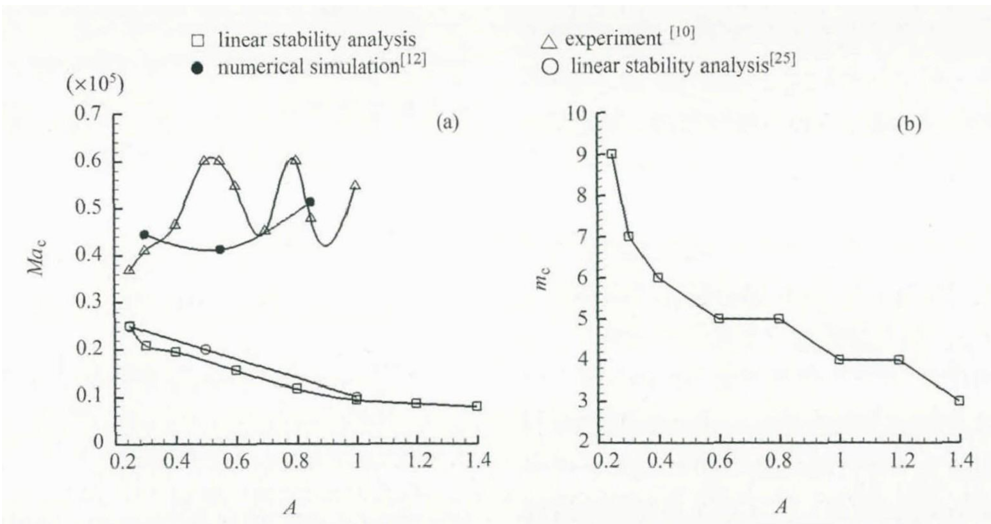

图 2 不同深宽比下的临界 Ma 数 (a) 与临界波数 (b)

Fig. 2 Critical Ma number (a) and and critical wave number (b) with various aspect ratio

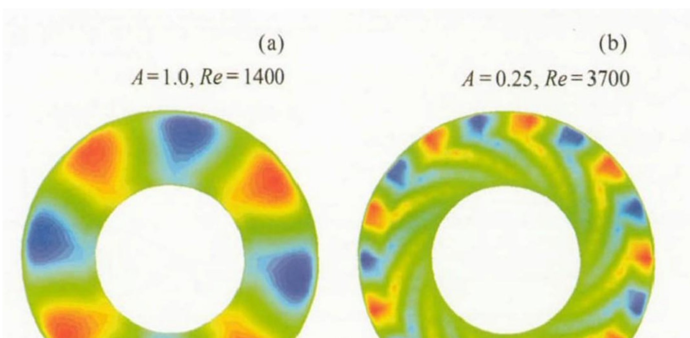

图 3 自由面扰动温度分布

Fig. 3 Distribution of temperature fluctuation on the free surface

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表 1 不同深宽比下扰动能量增长中的各参数变化

Table 1 Parameters in perturbation energy growth changing with various aspect ratio

A	m	Re	σ	M_r	M_θ	I_v
0.25	9	3700	0.24 + 94.65i	0.503 64	0.462 39	0.034 10
0.3	7	3100	0.42 + 62.21i	0.471 29	0.494 85	0.034 46
0.4	6	2900	0.16 + 41.25i	0.360 03	0.587 35	0.053 87
0.6	5	2300	0.14 + 29.87i	0.242 21	0.661 74	0.095 00
0.8	5	1800	0.41 + 24.21i	0.212 86	0.676 12	0.110 41
1.0	4	1400	0.15 + 18.66i	0.262 58	0.654 11	0.071 42
1.2	4	1300	0.27 + 17.21i	0.260 63	0.656 53	0.067 74
1.4	3	1200	0.20 + 15.35i	0.322 76	0.564 08	0.061 13

基准进行归一化处理.

表 1 列出了不同深宽比下扰动能量增长中的各项变化, 特征值中的 i 表示虚数单位. 由表 1 可以看出, 各项随液池深度的变化规律并不单调. 随着深宽比的增大, M_r 先减小后增大, M_θ 与 I_v 先增大后减小. 在 $A = 0.8$ 附近, 三项同时达到极值, 其中 M_r 达到极小值, 而 M_θ 与 I_v 达到极大值. 对比三者大小, 可以看出相对于表面张力做功项, 扰动流与基本流间的相互作用项较小. 不过在此 Pr 数下, 扰动流与基本流的相互作用项不能忽略.

4 结论

综上对环形液池热毛细对流线性失稳情况的分析, 可以得出如下主要结论.

(1) 线性稳定性分析预测的临界 Ma 数较小. 临界 Ma 数与临界波数均随深宽比的增大而减小.

(2) 能量分析结果表明, $Pr = 6.8$ 时扰动流与基本流间的相互作用项较小, 表面张力在径向做功与周向做功对小扰动的动能变化起主导作用.

(3) 观察扰动能量增长中的各项与液池深宽比的关系, $A = 0.8$ 时表面张力在径向做功项达到极小值, 在周向做功项及小扰动与基本流相互作用项达到极大值.

参考文献

[1] DERBY J J, KWON Y I, PANDY A, *et al.* Developing quantitative, multiscale models for microgravity crystal growth [J]. *Ann. N. Y. Acad. Sci.*, 2006, **1077**(1): 124-145

[2] SMITH M K, DAVIS S H. Instabilities of dynamic thermocapillary liquid layers Part 1. convective instabilities [J]. *J. Fluid Mech.*, 1983, **132**: 119-144

[3] SCHATZ M F, NEITZEL G P. Experiments on thermocapillary instabilities [J]. *Ann. Rev. Fluid Mech.*, 2001, **33**(1): 93-127

[4] KAMOTANI Y. Thermocapillary flow under microgravity—experimental results [J]. *Adv. Space Res.*, 1999, **24**(10): 13571366

[5] KAMOTANI Y, OSTRACH S. MASUD J. Microgravity experiments and analysis of oscillatory thermocapillary flow in cylindrical containers [J]. *J. Fluid Mech.*, 2000, **410**: 211-233

[6] MUKOLOBWIEZ N, CHIFFAUDEL A, DAVIAUD F. Supercritical Eckhaus instability for surface-tension-driven hydrothermal waves [J]. *Phys. Rev. Lett.*, 1998, **80**(21): 4661

[7] GARNIER N, CHIFFAUDEL A. Two dimensional hydrothermal waves in an extended cylindrical vessel [J]. *Eur. Phys. J.: B*, 2001, **19**(1): 87-95

[8] BENZ S, SCHWABE D. The three-dimensional stationary instability in dynamic thermocapillary shallow cavities [J]. *Exp. Fluids*, 2001, **31**(4): 409-416

[9] SCHWABE D, BENZ S. Thermocapillary flow instabilities in an annulus under microgravity—results of the experiment MAGIA [J]. *Adv. Space Res.*, 2002, **29**(4): 629-638

[10] SCHWABE D, ZEBIB A, SIM B C. Oscillatory thermocapillary convection in open cylindrical annuli Part 1. Experiments under microgravity [J]. *J. Fluid Mech.*, 2003, **491**: 239-258

[11] SIM B C, ZEBIB A, SCHWABE D. Oscillatory thermocapillary convection in open cylindrical annuli Part 2. Simulations [J]. *J. Fluid Mech.*, 2003, **491**: 259-274

[12] LI Yourong, PENG Lan, AKIYAMA Y, *et al.* Three-

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dimensional numerical simulation of thermocapillary flow of moderate Prandtl number fluid in an annular pool [J]. *J. Cryst. Growth*, 2003, **259**(4): 374-387

[13] LI Yourong, IMAISHI N, AZAMIC T, *et al.* Three-dimensional oscillatory flow in a thin annular pool of silicon melt [J]. *J. Cryst. Growth*, 2004, **260**(1): 28-42

[14] SHI Wanyuan, IMAISHI N. Hydrothermal waves in differentially heated shallow annular pools of silicone oil [J]. *J. Cryst. Growth*, 2006, **290**(1): 280-291

[15] PENG Lan, LI Yourong, SHI Wanyuan, *et al.* Three-dimensional thermocapillary-buoyancy flow of silicone oil in a differentially heated annular pool [J]. *Int. J. Heat Mass Trans.*, 2007, **50**(5): 872-880

[16] SCHWABE D. Buoyant-thermocapillary and pure thermocapillary convective instabilities in Czochralski systems [J]. *J. Cryst. Growth*, 2002, **237-239**: 1849-1853

[17] SHI Wanyuan, ERMAKOVC M K, IMAISHI N. Effect of pool rotation on thermocapillary convection in shallow annular pool of silicone oil [J]. *J. Cryst. Growth*, 2006, **294**(2): 474-485

[18] SHI Wanyuan, IMAISHI N. Hydrothermal waves in rotating annular pools of silicon melt [J]. *Microg. Sci. Technol.*, 2007, **19**(3): 159-160

[19] SHI Wanyuan, IMAISHI N. Thermocapillary convection in a shallow annular pool heated from inner wall [J]. *Microgravity Sci. Technol.*, 2007, **19**(3): 104-105

[20] SHI Wanyuan, LI Yourong, ERMAKOV M K, *et al.* Stability of thermocapillary convection in rotating shallow annular pool of silicon melt [J]. *Microgravity Sci. Technol.*, 2010, **22**(3): 315-320

[21] LAVALLEY R, AMBERG G, ALFREDSSON H. Experimental and numerical investigation of nonlinear thermocapillary oscillations in an annular geometry [J]. *Eur. J. Mech.: B Fluids*, 2001, **20**(6): 771-797

[22] LI Yourong, LIU Yushan, SHI Wanyuan, *et al.* Stability of thermocapillary convection in annular pools with low Prandtl number fluid [J]. *Microgravity. Sci. Technol.*, 2009, **21**(1): 283-287

[23] SHI Wanyuan, ERMAKOV M K, LI Yourong, *et al.* Influence of buoyancy force on thermocapillary convection instability in the differentially heated annular pools of silicon melt [J]. *Microgravity. Sci. Technol.*, 2009, **21**(1): 289-297

[24] SHI Wanyuan, LIU Xi, LI Guoyuan, *et al.* Thermocapillary convection instability in shallow annular pools by linear stability analysis [J]. *J. Supercond. Novel Magn.*, 2010, **23**(6): 1185-1188

[25] SHI Wanyuan, LI Yourong, ERMAKOV M K, *et al.* Linear stability analysis of thermocapillary convection in the annular pools of silicone oil [J]. *J. Eng. Thermophys.*, 2008, **29**(7): 1218-1220 (石万元, 李友荣, ERMAKOV M K, 等. 环形液层内热毛细对流的线性稳定性分析 [J]. 工程热物理学报, 2008, **29**(7): 1218-1220)

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