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DETERMINING THE NONLOCAL PARAMETERS OF A MICRO/NANOSTRUCTURE BY THE SHIFTS OF RESONANT FREQUENCIES

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As a structure size scales down to the order of a micron or smaller, there can be a significant changes in the mechanical properties as compared to its macroscopic ones. The nonlocal theories were developed to explain those changes, in which several material constants are involved. Those material constants are (assumed to be) related with the lattice length, grain size, inclusion and defects such as crack, dislocation and void, etc. However, there is still no a clear physical picture for those constants. Various experiments have been conducted to determine those material constants, which always use several specimens and often result in controversial data. Here we propose to use only one specimen and the shifts of its resonant frequencies to determine those material constants. The mechanical properties of a submicron structure is very sensitive to the defects, which is a main reason for the inconclusive results of the above experiments with several specimens. The presence of the material constants can have significant influence on the micro/nanostructure resonant frequencies. By measuring the shifts of resonant frequencies for one specimen, we determine those material constants by solving an inverse problem. Physically, the inverse problem can be solved because the material constants impact on one resonant frequency differently and the shifts of different resonant frequencies are different to one another. This inverse problem solving method gives not only a new but also a more reliable approach of experimentally determining the material constants.

1. Introduction

The nonlocal effects are due to the discrete and long-ranged nature of inter-atomistic/molecular interactions [1-6]. Because of the nonlocal effects, the mechanical behaviours, such as deformation and vibration, of nano/micrometer-scaled devices or even macroscopic ones, can significantly deviate from what are predicated by the classical local theories [1-6]. The material constants are the parameters determining the nonlocal effects. The full utility of the nonlocal theories hinges on one's ability to determine those intrinsic material constants. So far, there are very few works on determining the material constants and still no clear physical picture for them [5, 6]. To determine the material constants by the shifts of the device resonant frequencies forms an inverse problem [7, 8]. Determining the material constants is not only an important and complementary part of the nonlocal theories' development, but also of great help to the applications of nano/micrometer-scaled devices.

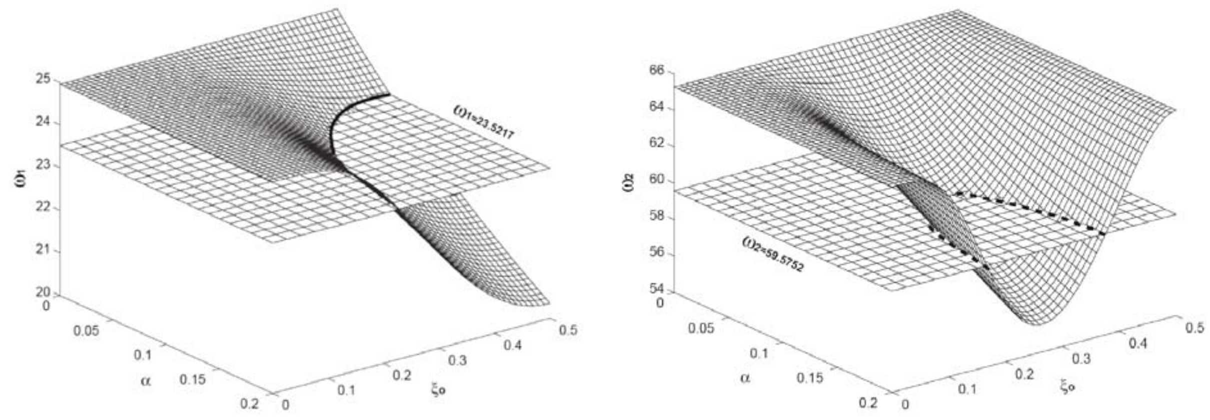
2. Model development and solution method

The following governing equation for an Euler-Bernoulli beam with the nonlocal effect is derived as follows [9]:

$$EIl_1^2 \frac{d^6 w}{dx^6} - (EI - l_2^2 \sigma_o A) \frac{d^4 w}{dx^4} - (l_2^2 \rho A \omega^2 + \sigma_o A) \frac{d^2 w}{dx^2} + \rho A \omega^2 w = 0. \quad (1)$$

Here E and ρ are the beam Young's modulus and density; I, A and σ_o are the beam moment of inertial, cross-section area and initial axial stress, respectively. w and ω are the beam displacement and the resonant frequency, respectively. l_1, l_2 are two (unknown) length parameters, which determine the nonlocal effects. It is noticed that now the governing equation is six order differential equation. When $l_1 = l_2 = 0$, clearly, Eq. (1) recovers the classical 4th order Euler-Bernoulli beam vibration equation we often encounter. However, with the presence of these two nonzero parameters, the beam vibration can be quite different from the one as predicated by the classic Euler-Bernoulli beam theory. As mentioned above, the presence of these two parameters are due to the discrete and long-ranged nature of inter-atomistic/molecular interactions. Here the nonlocal effects are embodied by these two parameter, which stand out when the specimen size is small. Physically, these two parameters are related with the grain size, twin and defects such as dislocation, void etc. So far, there is still no a clear physical picture on these two parameters. It is extremely difficult if not impossible to derive an equation to link the two parameters with the material properties and defects though the nonlocal effects have often been observed in various experiments.

The basic idea of determining the two parameters is to solve the inverse problem. Eq. (1) describes a continuous system with infinite resonant frequencies, which are also measurable in experiments. Usually, the resonant frequencies are computed as a forward/direct problem after the parameters including l_1, l_2 are supplied. As for inverse problem, the resonant frequencies are measurable and known quantities; l_1, l_2 are unknown. In logic terms, resonant frequencies are the results and l_1, l_2 are the causes. To demonstrate this inverse problem solving process, we give the example of a similar scenario: solving the inverse problem of the force and mass sensing in a resonantor [7]. In this scenario, there are actually three unknown parameters: the adsorbate mass, location and induced force. To make it simple and presentable in figure, here we reduce the three parameters to two: the adsorbate mass and location. The induced force is a fixed/given parameter. In Fig. 1, ω_1 and ω_2 are the dimensionless first and second resonant frequencies of the beam; α and ξ_o are the dimensionless adsorbate mass and location; β is the dimensionless induced force, which is fixed as β=10 for simplicity and illustration reason. To test the method of using the resonant frequencies to solve the inverse, we firstly set (α, ξ_o) = (0.1, 0.3), which then leads to ω_1 = 23.5217 and ω_2 = 59.5752. In the inverse problem, ω_1 and ω_2 are the known quantities and α, ξ_o are unknown. In Fig.1, the combinations of α and ξ_o on the first two resonant frequencies are presented. As in both Fig. 1 (a) and (b), the intersections are both curves, which physically means that for a given resonant frequency, there are infinite combinations of α and ξ_o. Therefore, one resonant frequency is not sufficient to determine two parameters.



(a) (b)

Figure 1: (a) The variation of the first resonant frequency (ω_1) as a function of α and ξ_o. The level plane is the one with the constant of ω_1 = 23.5217. The intersection of the two planes is marked with a solid curve. Here, the axial load is fixed as β = 10.

(b) The variation of the second resonant frequency (ω_2) as a function of α and ξ_o. The level plane is the one with the constant of ω_2 = 59.5752. The intersection of the two planes is marked with a dashed curve. Here, the axial load is fixed as β = 10.

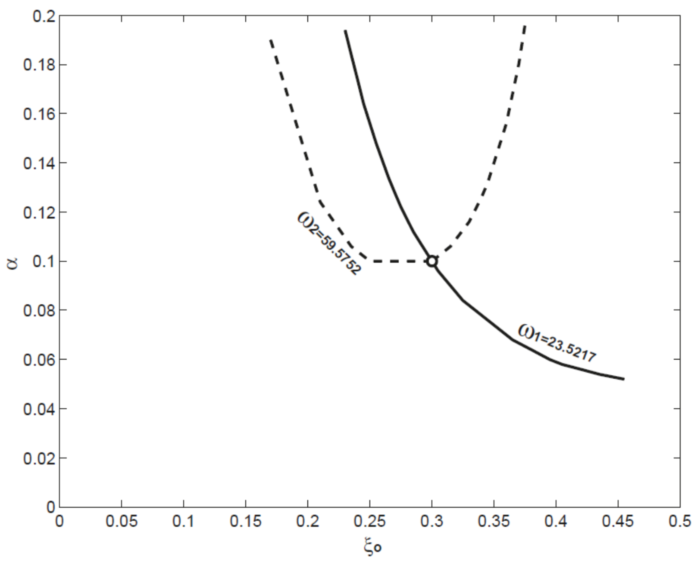


Figure 2: The projections of the two intersection curves obtained in Fig. 1 (a) and (b) into the α - ξ_o plane. The intersection of the two curves is marked with a circle, which corresponds to (α, ξ_o) = (0.1, 0.3) exactly.

However, if the two curves obtained in Fig. 1 are projected into the α - ξ_o plane, they intersect. As shown in Fig. 2, the intersection is marked with a circle, which corresponds to (α, ξ_o) = (0.1, 0.3) exactly. The inverse problem is thus solved. In the above example, the inverse problem can be solved because of the following two physical mechanisms: (1) different parameters have different impacts on one resonant frequency; (2) one parameter has different impacts on different resonant frequencies. Although there are some difference between the governing equations of the above scenario of adsorbate and the nonlocal model, the above two mechanisms are believed to be still applicable. As seen in Eq. (1), the two parameters appear differently in the equation, which implies different physical impacts on resonant frequencies.

3. Summary

Using the resonant frequencies to determine the unknown parameters in the governing equation of a beam structure is demonstrated. Because there are infinite resonant frequencies in a continuous system, we can provide *N* resonant frequencies for *N* unknown parameters. However, one thing we should keep in mind is that most inverse problems are not well-posed problems. In the above example, the inverse problem of two parameters is solved by two resonant frequencies, which may not be the general case. In some cases, resonant frequencies more than two are required to solve the inverse problem of two parameters.

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