An energy integral method analysis of preferred modes and the spreading rates of supersonic circular jets

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A linear frequency response of compressible axisymmetric jets to an external periodic perturbation of real frequency at the jet exit was analyzed based on energy integral method. The focus of the analysis is on the dependence of jet-preferred mode, characterized by the non-dimensional Strouhal number St_p , on the jet parameters, such as the Reynolds number, the amplitude of perturbation, the momentum thickness, and particularly the jet Mach number M_0, at the jet exit. The results show that St_p decreases first with increasing M_0 from zero because the suppression effect of flow compressibility on the growth of high-frequency perturbations is stronger than that of low-frequency ones. As M_0 increases to above a critical value, the suppression is of approximately the same extent to all the perturbations. Consequently, St_p starts to increase because the jet development favors high-frequency perturbations, which have higher rates of energy transfer from the base jet flow and hence larger growth rates. With further increasing M_0 , St_p decreases again mainly because the viscous dissipation suppresses the growth of high-frequency perturbations more significantly than that of low-frequency ones. The influences of the Reynolds number, the amplitude of perturbation, and the presumed length of potential core were investigated to substantiate the non-monotonic variation of St_p with M_0. The present theoretical results were found to agree well with available experimental results and the existing discrepancies were discussed.

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$$\hat{u} = \frac{1}{\bar{\rho}_s(\bar{u}_s\alpha - \omega)}\left(\frac{d\bar{u}_s}{dr}\frac{d\hat{p}}{dr}\frac{1}{\bar{u}_s\alpha - \omega} + \alpha\hat{p}\right) \quad (A5)$$

$$\hat{v} = \frac{i}{\bar{\rho}_s(\bar{u}_s\alpha - \omega)}\frac{d\hat{p}}{dr} \quad (A6)$$

at the both boundaries of $r = 0$ and $r = \infty$, one has the relation

$$\frac{1}{\Omega}\frac{d\Omega}{dr} \to 0 \quad (A7)$$

so that the Rayleigh can be asymptotically approximated by the modified Bessel equation

$$\frac{d^2\hat{p}}{dr^2} + \frac{1}{r}\frac{d\hat{p}}{dr} - \alpha^2(1 - M^2\Omega)\hat{p} = 0 \quad (A8)$$

The asymptotic solutions of (A6) are

$$\hat{p}(r \to 0) \sim C_1 I_0\left(\alpha\sqrt{1 - M^2\Omega(0)}r\right) \quad (A9)$$

and

$$\hat{p}(r \to \infty) \sim C_2 K_0\left(\alpha\sqrt{1 - M^2\Omega(\infty)}r\right) \quad (A10)$$

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