Lateral migration of fiber in settling suspensions

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Abstract

When fiber suspensions settle down in a headbox of a paper machine by gravity, the suspensions may create a weak vertical shear flow in the vicinity of the suspensions due to the variation of fiber number density. A fiber near the suspension may experience a shear force due to the vertical stream lines and migrate laterally either toward the high particle number density region or toward the lower particle density region depending on the settling Reynolds number. Such the cross-stream or lateral migration of a single fiber settling in a bounded weak shear flow with vertical streamlines, produced by a perturbation to the fiber number density, is studied using lattice Boltzmann simulations. The present simulation results show that at a given shear rate, the lateral migration can be divided into three phases depending on settling Reynolds number R_{sd} and fiber aspect ratio κ. When κ is given, at a low settling Reynolds number R_{sd} the shear force may dominate the fiber rotational inertia and the fiber will tumble and migrate toward the lower fiber density region and the fiber is finally stopped in the cross-stream direction by the wall repulsive force. This is phase one where the suspension becomes more stable. As R_{sd} increases and exceeds a critical settling Reynolds number R_{sd1}, the fiber attempts to align its long body with the horizontal direction due to inertial torque. On the other hand, the torque due to the imposed weak vertical shear flow rotates the non-spherical fiber in the opposite direction. The dynamic balance between the two torques may lead to a small angle between the fiber long body and horizontal plane, which may drive fiber migrate toward the higher fiber density region. This is phase two where the suspension is destabilized. This mechanism recently proposed by Shin, Koch and Subramanian 2009 is examined and confirmed by the present simulation results. As the settling Reynolds number continuously increases and exceeds a second critical R_{sd2}, the orientational inertia dominates the imposed vertical shear force and the fiber will remain in the center between two wall on time average. This is phase three.

1 Introduction

Sedimentation is popularly used to separate solid particles from fluids and to classify the different particles in term of their sizes by chemical engineers. For instant, in the paper industry, fines can be separated from long fibers by sedimenting process since fines have huge surface areas, receive larger drag and settle slowly.

When fiber suspensions settle down vertically in a headbox of a paper machine by the gravity, the suspensions may create a weak vertical shear flow in the vicinity of the suspensions due to the variation of fiber number density. A fiber near the suspension may experience a shear force due to the vertical stream lines and migrate laterally either toward the high particle number density region or toward the lower particle density region depending on the settling Reynolds numbers $R_{sd} = V_z l/\nu$, where V_z is the vertical settling velocity of the mass center of the fiber, l is the half fiber length, ν is the kinematic viscosity. As compared with R_{sd}, the induced vertical shear is very weak, $R_s = Gl^2/\nu << 1$ where G is the shear rate. When the fiber migrates toward the high number density region, suspension become more inhomogeneous and unstable. This important problem has been investigated by many authors. In the Stokes flow regime, Koch and Shaqfeh[3] showed that the hydrodynamic disturbance of one fiber will rotate a neighboring fiber to spend more time in the vicinity of its neighbors. They showed that a homogeneous suspension of fibers is unstable and that waves of fiber concentration with horizontal wave vectors create flows that orient fibers migrating toward the higher fiber number density regions. Experiments and numerical simulations [4, 5, 6, 7, 8, 9] have confirmed the finding of Koch and Shaqfeh.

To understand stability of suspension, how the lateral migration of a settling fiber responds to the vertical shear flow was analyzed by using slender body theory at small Reynolds numbers. There are two mechanisms attributed to the lateral migration of a finite Reynolds number fiber in a simple vertical shear flow. First, the inertial coupling of the fluid velocity fields with the imposed shear fluid can create a cross-stream lift. This has been reported for spherical particles through asymptotic analysis for small Reynolds number and numerical simulations for finite Reynolds numbers. A spherical particle at very small Reynolds number will migrate toward streamlines of the imposed flow field with smaller flowing velocity originally found by Saffman [11]. The lift will drive spherical particle to migrate toward a lower particle density area and make the suspension more stable. This mechanism also persists for non-spherical particles. The second mechanism was recently proposed by Shin et al[10] who conducted a slender-body analysis in a simple shear flow under the condition of $R_s/R_{sd} << 1$. They indicated that a fiber at finite Reynolds number attempts to align its long axis with horizontal direction due to nonlinear inertial torque while the torque due to the imposed vertical weak shear rotates fiber in the opposite direction. The balance between the two torques results in an inclined angle between the fiber axis and the horizontal direction and drives the fiber migrate toward high number density region, due to the force of the gravity and the anisotropy of its resistance to translational motion. The second mechanism only exists in a non-spherical particle suspension. Certainly, the combination of the two mechanisms will determine the final net cross- stream or lateral migration or in turn determine the stability of suspensions.

It seems that a separation technique may be developed to separate fibers depending on different lateral migration direction and velocity by using different settling Reynolds numbers and using a weak shear flow with vertical streamlines. A quantitative description of how the Reynolds number and fiber aspect ratio affect lateral migration direction and speed is necessary before designing and building a separation device in a lab.

The present work will conduct a series of numerical simulations of a single sedimenting fiber in a bounded weak vertical shear flow by using lattice Boltzmann method. The mechanism proposed by Shin et al. will be numerically examined and the influence of the imposed weak vertical shear flow on lateral migration at different values of R_{sd} and κ will be investigated. Section 2 presents the simulation conditions; section 3 will report the simulation results of the cross- steam migration at different R_{sd}, R_s and aspect ratio . It is found that the migration can be classified into three different phases. Section 4 gives some conclusion.

2 Simulation parameters

A vertical shear fluid flow is initially set at the left wall with velocity $V_z = 0$ at $y = 0$ and the right wall with velocity $V_z = U_0$ at $y = N_y$. The gravity is in the z-direction, shear gradient in the y-direction and vorticity in the x-direction as shown in figure 1. A prolate spheroidal fiber with a half length of $l = 0.5L$ is used, where L is the fiber length, and its aspect ratio may be varied at different aspect ratios κ in the simulations. The simulations are carried out in a box of $64 \times 64 \times 128$ where the confinement ratio is $C = L/N_y = 0.375$. The fiber initially settles with its principal z'-axis parallel to the horizontal direction. The lateral or crossing stream migration and rotation are allowed. In a short time period, a fiber will migrate in the y-direction and rotate around its short axis only that is parallel to the x-axis. This rotation is limited to the vertical yz-plane. The rotational angle θ between the fiber principal z'-axis and the vertical direction or z-axis can be defined. Most simulations in the present work are limited to the short period time.

The lattice Boltzmann method with 3DQ15 model is adopted[13]. The method has been reported [14, 15, 16, 17, 18] previously and will not be repeated here. The length is normalized by the width N_y and the velocity is normalized by $U_0 = 10N_yG$ for this report

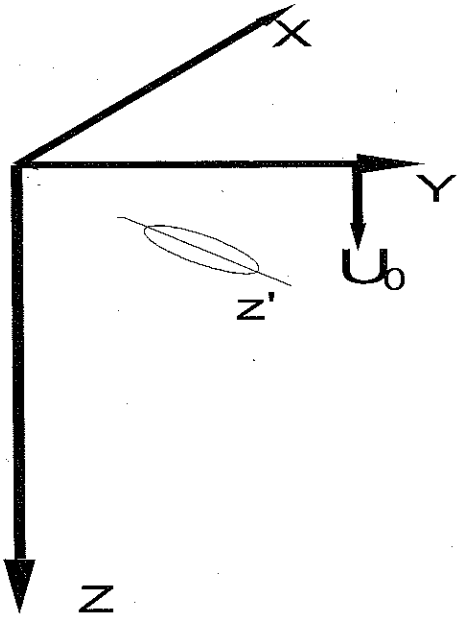

Figure 1: A shear velocity $V_z = U_0$ is imposed on the right wall at $y = N_y$ and $V_z = 0$ is imposed on the left wall at $y = 0$. The vorticity of the weak shear is in the x-direction, the gravity is in the z-direction. θ (not shown) is the angle between the fiber z'-axis and z-axis and the ϕ (not shown) is the angle between the y-axis and the projection of the fiber z'-axis on the xy-plane.

where the shear rate $G = 0.000111$.

3 Simulation results

The different settling Reynolds numbers R_{sd} are obtained by varying solid density ρ_s and kinematic viscosity ν while the time and length units are kept same. It is found that at a given shear Reynolds number R_s, fiber lateral migration can be divided into three different phases depending on the settling Reynolds number R_{sd} and fiber aspect ratio κ.

At a low settling Reynolds number, nonlinear inertial effects are very weak and the imposed vertical shear may dominate the inertia. The shear force drives the fiber to rotate and tumble continuously (see figure 3 (b)) and migrate with the time average velocity $< V_y >< 0$ as shown in figure 2 for its lateral migration and velocity and in figure 3 for its vertical velocity and fiber orientational angle θ for the case of $R_s = 0.1$ and $\kappa = 1.6$. The relative velocity of the fluid flow to the fiber in its left is larger than that in the right and the generated pressure difference between the two sides forces the fiber to migrate with $< V_y >< 0$. Finally the lateral migration is stopped, on time average, near the left wall until the lift force is balanced by the wall repulsion. This type of force balance is similar to Segre and Silberbergi phenomena [19, 20]. This is called phase one. It seems that the lateral migration increases with R_{sd} decreases in phase one.

As R_{sd} increases and becomes larger than a critical value$R_{sd} > R_{sd1}$, the inertia become larger enough and the fiber attends to be aligned with the horizontal direction. The inertial alignment may be balanced by the imposed weak shear, a small angle between the horizontal direction and fiber axis is created and causes fiber to migrate with $V_y > 0$ until it is stopped in the lateral-direction by wall repel force as shown in figure 4 (a) for its lateral displace and velocity, settling velocity and orientational angle. In other words, the fiber moves to the higher fiber number density region, inhomogeneous is enhanced by such the migration. This is called phase 2.

It is noted that there are three different cases in phase 2. In the first case, at a lower settling Reynolds number, for instant, at $R_{sd} = 1.028$, the fiber always rotates and tumbles as shown in figure 4 (a) for the migration and (d) for the rotational angle. However, on time average, the fiber migrates toward the right wall. In the second case, as the settling Reynolds number increases, the fiber initially oscillates and finally becomes stable such as the case of $R_{sd} = 1.203$. This oscillation case is consistent with Huang et al.'s observation [21]. In the third case, the lateral migrations of the fiber at $R_{sd} = 1.74$, 2.35 and 3.51 have

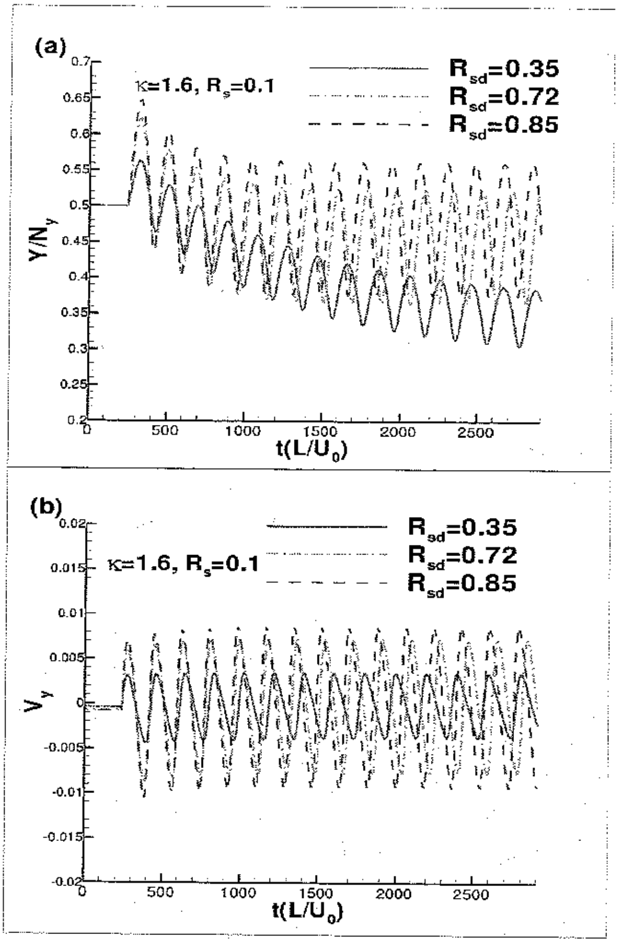

Figure 2: (a) Lateral migration displacement and (b) lateral velocity of fiber as a function of time at different settling Reynolds numbers in the phase 1 for the case of $R_s = 0.1$ and $\kappa = 1.6$

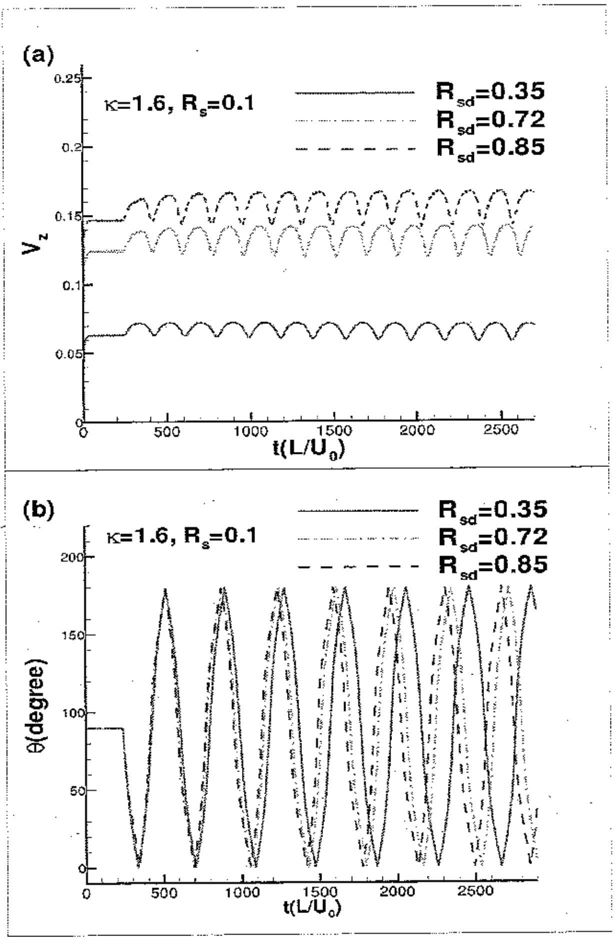

Figure 3: (a) The fiber vertical velocity and (b) orientational angle θ as a function of time at different settling Reynolds numbers in the phase 1 for the case of $R_s = 0.1$ and $\kappa = 1.6$

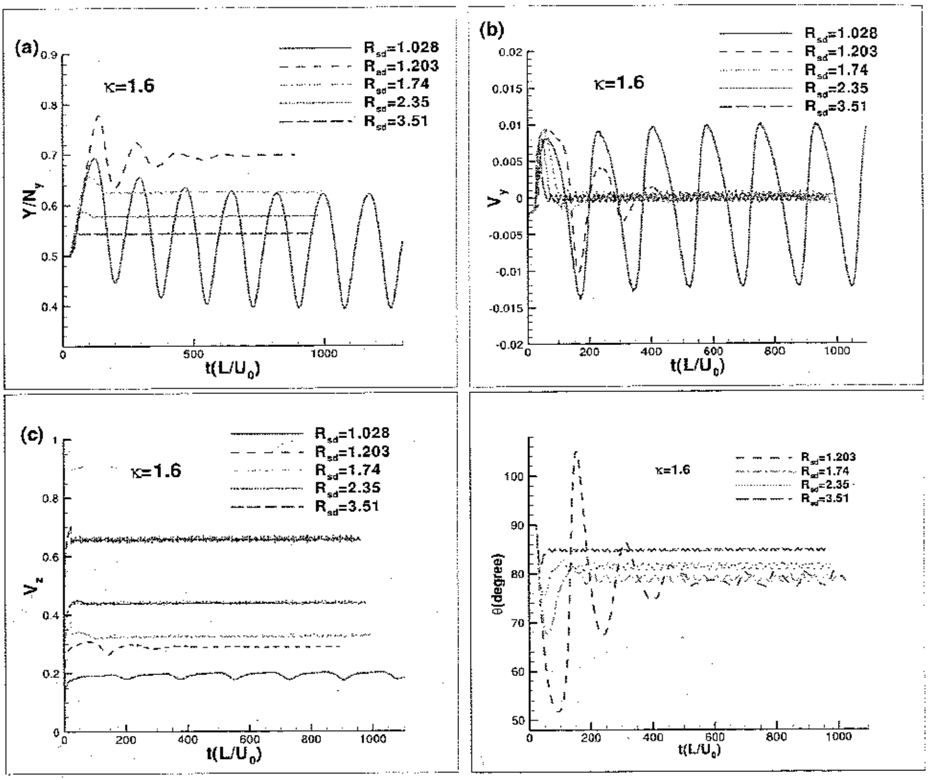

Figure 4: (a)Lateral migration displacement, (b) lateral velocity, (c) vertical velocity and (d) orientational angle as a function of time at different settling Reynolds numbers R_{sd} in the phase 2 for the cases of $\kappa = 1.6$.

no oscillation. In general, it is shown that the lateral migration reduces as R_{sd} increases due to the angle gradually approaching to 90^0.

As R_{sd} increases continuously until it is larger than a 2nd critical number $R_{sd} > R_{sd2}$ the fiber inertial alignment may dominate the weak shear and maintains its long axis horizontally. Thus, the fiber may remain in the center between two walls. This is phase 3. At a large settling Reynolds number, although the wake effect may induce oscillation of both orientational angle and lateral migration around their corresponding equilibrium position, the lateral migration diminishes on average of time. It seems that the settling Reynolds number has little effect on the lateral migration as shown in figure 5 for the lateral migration and velocity and the vertical velocity and orientational angle. When R_{sd} increases from 24.02 to 44.51, although the amplitude of oscillation of both the lateral displacement and orientation angle increases, the average lateral migration does not change too much, indicating that the lateral migration in phase 3 is clearly independent from R_{sd}. The oscillation is caused by the wake effect or vortex behind the fiber. For the case of $R_{sd} = 44.51$, the estimated Strouhal number $St = 0.198(1 - 19.7/R_{sd})$ is around 0.026. It is noted that the vortex shedding does not affect the lateral migration.

4 Conclusions

A series of the lattice Boltzmann simulations are conducted to investigate lateral migration of a fiber settling in a weak vertical shear flow. In the simulations, a single prolate spheroidal fiber is settling in a vertically bounded linear weak shear field. The fiber receives a lift force and migrates laterally. Since the imposed shear field represents the influence of motion of other fiber suspensions and is driven by horizontal inhomogeneities in the fiber concentration, the direction of the cross-stream or lateral migration is directly related to the stability of the suspensions. The simulation results demonstrate that at a given shear rate and a given

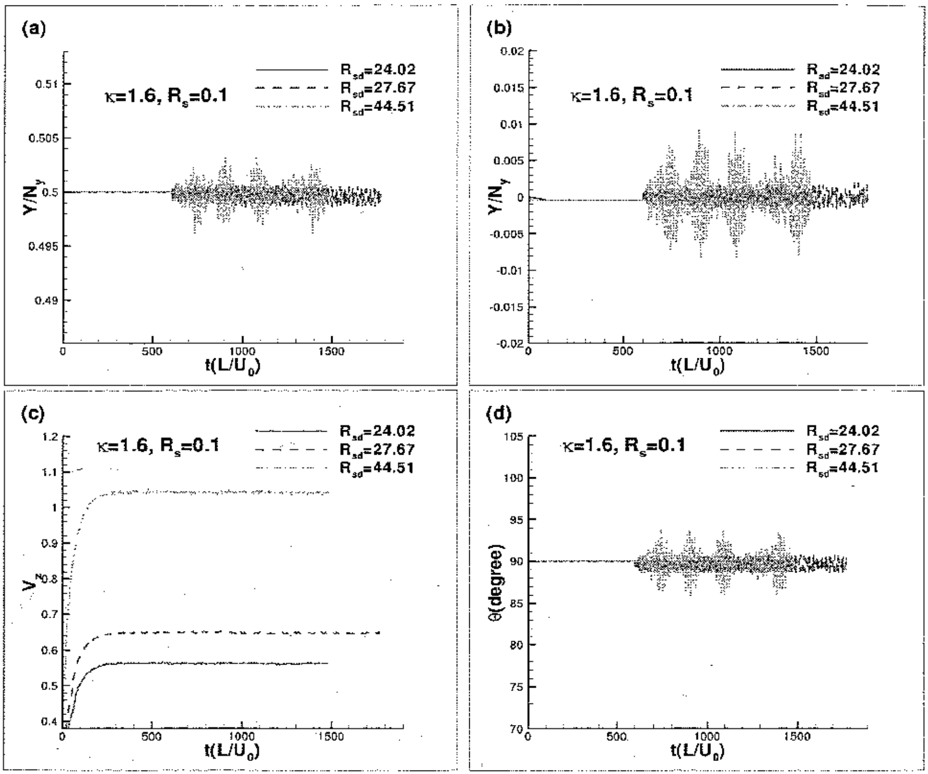

Figure 5: (a)Lateral migration displacement, (b) lateral velocity of fiber (c) vertical velocity and (d) orientational angle as a function of time at different settling Reynolds numbers for the case of $\kappa = 1.6$ in phase 3.

particle aspect ratio, the cross-stream or lateral migration can be divided into three phases. At a low settling Reynolds number, the shear force may dominate the inertia and the fiber may tumble and migrate with $< V_y >< 0$. This is phase 1 where fiber suspension becomes more homogeneous and stable. As Reynolds number R_{sd} increases and exceeds a critical Reynolds number R_{sd1}, the fiber aligns its long axis with the horizontal direction due to inertial torque that may be balanced by the imposed shear force, resulting an inclined angle between the fiber axis and horizontal plane and driving a lateral migration with $V_y > 0$. This is phase 2 where inhomogeneity is reinforced. As the settling Reynolds number continuously increases and exceeds the second critical number R_{sd2}, the inertia dominate the weak shear force, the fiber long axis is kept in the horizontal direction and may remain in the center between two walls. This is phase 3 where the fiber has little lateral migration and it is independent from Reynolds number. It is demonstrated that in phase 1 the lateral migration with $V_y < 0$ increases as the shear rate increase while in phase 2 the lateral migration with $V_y > 0$ reduces as the Reynolds number increases. In phase 3, shear rate has little effect on lateral migration. The property of lateral migration direction depending on Reynolds number when fiber settling in a weak vertical shear flow may be used to develop a device to separate fibers.

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