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Multiple Models Adaptive Switching Control Based on Feasible Controller Set

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***Abstract*—In this paper a new systematic switching control approach to adaptive stabilization of parameters uncertain linear systems is presented. One feature of this approach based on feasible controller set is its controller falsification capability, which is manifested as the rapid convergence of the switching controller, another feature is its capability of improving the close-loop transient response and reducing computation burden. In addition, the potential advantages of the presented approach include the applicability to both continuous and discrete uncertainty system and the simplicity of the stability analysis.**

Keywords- Feasible controller set; Multiple models; Switching control; Uncertain system

I. Introduction

In the traditional design of controllers for plant uncertainty, the general approaches often adapted are adaptive control and robust control. As far as adaptive control is concerned, if sudden change in plant dynamic happens due to operating environment, component failure or external influence, the transient response may be poor and system is even unstable. As for robust control (H_2, H_∞ and the like), they can only deal with the model of which uncertainties are "sufficiently small".

To cope with "large model uncertainties" and improve transient response, switching control is developed in recent years [1]-[9]. There are two kinds of switching control methods: (1) Evaluating every candidate controller's performance by applying it to the process in a predetermined sequence [1]-[4]; (2) orchestrating feedback controllers into the process of switching from a precomputed finite (continuum) set of fixed controllers based on certain online estimation [5]-[8]. The first method has the advantage of light computation burden, but the controller search may converge very slowly resulting in excessive transients which renders the system unstable in a practical sense; the second method can improve the system transient response, however the computation burden of this method is heavy when more models are considered and several issue still remain unresolved, these include the controller being nonconvergent and the proofs of stability being too complicated [5].

In this paper, a new class of adaptive switching mechanisms is presented to overcome the drawbacks mentioned above. This approach is based on the feasible controller set which incorporates simultaneous falsification of a number of controllers and therefore, improves controller converge rate and reduces online estimation computation burden. The potential advantages of this approach include the finite convergence for switching, the simplicity of the stability analysis and the applicability to both continuous and discrete system.

II. Problem Formulation and Basic Lemma

It is assumed that the uncertain plant to be controlled may be described by a model P_m contained in a finite set P of parameter uncertain continuous linear system models, and assumed that a finite set of robust controllers{ Π_i }, i=1,2,……s, $s \in N$, has been found so that for each plant $P_i \in$ P, i=1,2,……s, there exists one controller Π_i , i=1,2,……s, so that the resulting closed-loop system{P_i, Π_i} is stable, a switching control algorithm is proposed in this paper to select the real controller out of controller set { Π_i } with the property of stability of closed-loop system.
Where

$$P = \bigcup_{i=1}^{s} P_i$$

$$P_i : \begin{cases} \dot{x} = A_i x + B_{i1} u + B_{i2}\omega \\ y = C_{i1} x + D_{i12}\omega \\ z = C_{i2} x + D_{i21} u + D_{i22}\omega \end{cases} \quad (1)$$

$$A_i = A_{ni} + \Delta A_i \qquad i=1,2,\ldots\ldots s$$

Where A_{ni} is the constant matrix and ΔA_i is uncertain parameter. The others system parameter matrices have the same form as A_i with appropriate dimension and all uncertain parameters ($\Delta A_i, \Delta B_{i1}, \Delta B_{i2}, \Delta C_{i1}, \Delta C_{i2}, \Delta D_{i12}, \Delta D_{i21}$, ΔD_{i22}) satisfy norm-bounded parameter uncertainties.

$$\begin{bmatrix} \Delta A_i & \Delta B_{i1} & \Delta B_{i2} \\ \Delta C_{i2} & \Delta D_{i21} & \Delta D_{i22} \end{bmatrix} = \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix} F \begin{bmatrix} N_{1i} & N_{2i} & N_{3i} \end{bmatrix}$$

$$\begin{bmatrix} \Delta C_{i1} & \Delta D_{i12} \end{bmatrix} = M_{3i} F \begin{bmatrix} N_{1i} & N_{3i} \end{bmatrix} \quad i=1, 2,\ldots\ldots s$$

Where F is an unknown real matrix satisfying

$$FF^T \le I$$

$M_{1i}, M_{2i}, M_{3i}, N_{1i}, N_{2i}, N_{3i}$ are the known matrices with appropriate dimension.

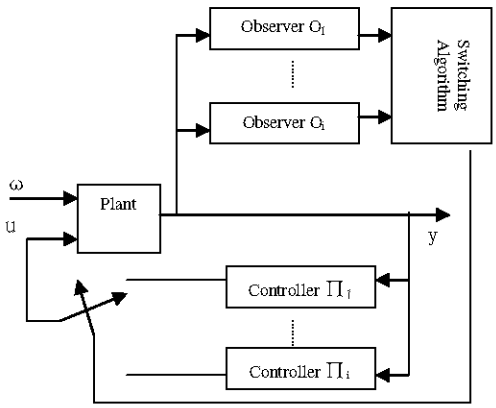

Figure 1 The schematic block diagram of switching control

A robust H_∞ controller Π_i is designed for each models P_i so that the resulting closed-loop system $\{P_i, \Pi_i\}$ is stable and all controllers compose controller set Π,

$$\Pi = \bigcup_{i=1}^{s} \Pi_i$$

$$\Pi_i : \begin{cases} \dot{\varepsilon} = A_i^{\Pi}\varepsilon + B_i^{\Pi} y \\ u = C_i^{\Pi}\varepsilon \end{cases} \tag{2}$$

i=1,2,……s

Where ε is controller state.

A robust H_∞ observer O_i is designed for each close-loop system $\{P_i, \Pi_i\}$ (the model P_i and its controller Π_i), i=1, 2, ···s.

$$O_i : \begin{cases} \dot{\eta} = A_i^{o}\eta + B_i^{o} y \\ y_{io} = C_i^{o}\eta + D_i^{o} y \end{cases} \tag{3}$$

i=1,2,……s

Where η is observer state, y_{io} is observer output.

Remark 2.1: The robust H_∞ observer O_i can not be designed for an unstable parameters uncertain system as (1), it is designed only for the stable system $\{P_i, \Pi_i\}$.

Assume that $P_m \in P$, $1 \le m \le s$ is the real plant model.

$$P_m : \begin{cases} \dot{x} = A_m x + B_{m1} u + B_{m2}\omega \\ y = C_{m1} x + D_{m12}\omega \\ z = C_{m2} x + D_{m21} u + D_{m22}\omega \end{cases} \tag{4}$$

The close-loop system that results on applying controller Π_i (i=1,2,…...s) to P_m is $\Pi_i P_m$,

$$\Pi_i P_m : \begin{cases} \underbrace{\begin{pmatrix} \dot{x} \\ \dot{\varepsilon} \end{pmatrix}}_{} = \underbrace{\begin{pmatrix} A_m & B_{m1} C_i^{\Pi} \\ B_i^{\Pi} C_{m1} & A_i^{\Pi} \end{pmatrix}}_{\bar{A}_i} \underbrace{\begin{pmatrix} x \\ \varepsilon \end{pmatrix}}_{x_\varepsilon} + \underbrace{\begin{pmatrix} B_{m2} \\ B_i^{\Pi} D_{m12} \end{pmatrix}}_{\bar{B}_i} \omega \\ y = \begin{bmatrix} C_{m1} & 0 \end{bmatrix} x_\varepsilon + D_{m12}\omega \\ z = \begin{bmatrix} C_{m2} & D_{m21} C_i^{\Pi} \end{bmatrix} x_\varepsilon + D_{m22}\omega \end{cases} \tag{5}$$

Combining (3) with (5), we obtain the following augmented system $\{\Pi_i P_m O_i\}$, i={1,2,……s}

$$\begin{cases} \begin{pmatrix} \dot{x}_\varepsilon \\ \dot{\eta} \\ \dot{e}_f \end{pmatrix} = \underbrace{\begin{pmatrix} \bar{A}_i & 0 & 0 \\ \begin{bmatrix} B_i^{o} C_{m1} & 0 \end{bmatrix} & A_i^{o} & 0 \\ \begin{bmatrix} \lambda(I - D_i^{o}) C_{m1} & 0 \end{bmatrix} & -\lambda C_i^{o} & -\lambda I \end{pmatrix}}_{\bar{\bar{A}}_i} \underbrace{\begin{pmatrix} x_\varepsilon \\ \eta \\ e_f \end{pmatrix}}_{\tilde{Z}} \\ + \underbrace{\begin{pmatrix} \bar{B}_i \\ B_i^{o} D_{m12} \\ \lambda(I - D_i^{o}) D_{m12} \end{pmatrix}}_{\bar{\bar{B}}_i} \omega \end{cases} \tag{6}$$

Where e_f is the filtered observer error e_i=y-y_{io} given by

$$\dot{e}_f(t) = -\lambda e_f(t) + \lambda e_i(t) \qquad \lambda > 0 \tag{7}$$

When the controller Π_i is applied to P_m, the filtered observer error e_j=y-y_{jo} of each observer O_j is obtained by augmented system $\{\Pi_i P_m O_j\}$, i,j=1,2,……s.

$$\dot{e}_{fj}(t) = -\lambda e_{fj}(t) + \lambda e_j(t)$$

$$\begin{cases} \begin{pmatrix} \dot{x}_\varepsilon \\ \dot{\eta}_j \\ \dot{e}_{fj} \end{pmatrix} = \begin{bmatrix} \bar{A}_i & 0 & 0 \\ \begin{bmatrix} B_j^{o} C_{m1} & 0 \end{bmatrix} & A_j^{o} & 0 \\ [\lambda(I - D_j^{o}) C_{m1}\ 0] & -\lambda C_j^{o} & -\lambda I \end{bmatrix} \begin{pmatrix} x_\varepsilon \\ \eta_j \\ e_{fj} \end{pmatrix} \\ + \begin{bmatrix} \bar{B}_i \\ B_j^{o} D_{m12} \\ \lambda(I - D_j^{o}) D_{m12} \end{bmatrix} \omega \end{cases} \tag{8}$$

Throughout this paper, the ∞-norm of x=[x_1,……,x_n] $\in$ R^m and the induced ∞ -norm of $X \in R^{m\times n}$ is denoted respectively as

$$\| x \| := \max_{i\in\{1,2,\ldots m\}} | x_i |$$

$$\| X \| := \max_{i\in\{1,2,\ldots m\}} \sum_{j=1}^{n} | X_{ij} |$$

As well, the L_∞ norm of a function $f : R^+ \cup \{0\} \to R^n$ is denoted as $\| f \| := \operatorname{ess\,sup}_{t>0} \| f(t) \|$

lemma2.1: if $\overline{\overline{A}}_i$ of (6) is Hurwitz matrix, and $\varpi(t)$ is piecewise continuous bounded signal having L_∞ norms of $\overline{\varpi}$, then there exist constants $\xi_1 > 0, \xi_2 > 0$, such that for all t>0,

$$\| \tilde{z}(t) \| \le \xi_1 \| \tilde{z}(0) \| + \xi_2 \quad (9)$$

Proof: See [1] lemma 1.

III. Switching Control Based on Feasible Controller Set

The presented switching control algorithm in this paper is based on the feasible controller set which incorporates simultaneous falsification of a number of controllers, therefore improves controller converge rate and reduces computation burden. The algorithm will determine the controller switching time and select the correct controller from the feasible controller set for an unknown uncertain plant P_m. The schematic block diagram of switching control is shown as Fig 1.

Definition 3.1: A function $f : N \to R$ is said to bounding function if it is strictly increasing and if for certain constants $c_0>0$, $c_1>0$,

$$f(p)/(c_0 + c_1 \sum_{i=1}^{p-1} f(i)) > 1 \quad (10)$$

Definition 3.2: A set $F(t_k)=\{\Pi_j\}$ is said to be feasible controller set at the kth switching time t_k if when the controller Π_i is applied to P_m, the augmented system $\{\Pi_i P_m O_j\}$ meets the following inequality at t_k,

$$\| \eta_j(t_k), e_{fj}(t_k) \| < b_0 + b_1 \sum_{i=1}^{k-1} f(i) \quad (11)$$

for certain constants $b_0>0$ and $b_1>0$, i.e:

$F(t_k)=\{\Pi_j \mid$ the augmented system $\{\Pi_i P_m O_j\}$ meets(11) according to (8)$\}$.

Remark 3.1: According to the definition of $F(t_k)$, if the state and the error of the observer O_j meet (11), the corresponding controller Π_j of O_j belongs to $F(t_k)$. That a controller Π_j is said to be feasible controller means that it is "possible" for Π_j to make the system stable, in other word, if a observer does not meet (11), its corresponding controller must not be real controller. See the property (1) of theorem 4.1.

Switching control algorithm for linear continuous systems:

Algorithm 3.1:

1). At t=initial time t_1, let the feasible controller set $F(t_1)=\{\Pi_1, \Pi_2, \ldots\ldots \Pi_s\}$.

2). For $t>=t_1$, apply any controller (for example Π_j) of the feasible controller set $F(t_{k-1})$ to plant at t_{k-1} (k>=2)and determine the switching time t_k by

$$t_k := \begin{cases} \min\{t \mid t > t_{k-1}, \| [\varepsilon(t)^T, \eta(t)^T, e_f(t)^T]^T \| = f(k-1)\} \\ \quad \text{if this minimumexist} \\ \infty \quad \text{otherwise} \end{cases}$$

according to (6).

3).If $t_k=\infty$, end. Otherwise modify $F(t_{k-1})$:

$$F(t_{k-1}) = F(t_{k-1}) / \Pi_j$$

4). Determine the feasible controller set $F(t_k)$ at t_k in $F(t_{k-1})$

$$F(t_k) = \{\Pi_n \mid \Pi_n \in F(t_{k-1}) \;\&\; \| [\eta_n(t_k)^T, e_{fn}(t_k)^T]^T \| < b_0 + b_1 \sum_{i=1}^{k-1} f(i)\}$$

according to (8).

5).k=k+1

6). Return 2).

Remark 3.2: The feasible controller set method is partly similar to the Localization method [9] which apply to only discrete system, the feasible controller set method, however, can apply to not only discrete system, but also continuous system.

IV. Main Results

Assumption 4.1:

(1) $\| [\varepsilon(t_1)^T, \eta(t_1)^T, e_f(t_1)^T]^T \| < f(1)$

(2) $\| [\eta_i(t_1)^T, e_{fi}(t_1)^T]^T \| < b_0 + b_1 f(1)$

i=1,2,…s

Theorem4.1. Consider a plant contained in P to which algorithm 3.1 is applied at time $t=t_1$ and assumption 4.1 holds, then for bounded piecewise continuous signal ϖ,bounding function f and certain positive constants b_0,b_1, c_0,c_1, the close-loop system has following properties that:

1).the state of the real observer O_m and the filtered real observer error satisfy:

$$\| [\eta_m(t_k)^T, e_{fm}(t_k)^T]^T \| < b_0 + b_1 \sum_{i=1}^{k-1} f(i)$$

at t_k. for suitable constants $b_0>0$, $b_1>0$.

2).there exist a finite time $t_j>=t_1$ and constant controller Π_i such that for $t>=t_j$, $\Pi(j) = \Pi_i$, i={1,2,……s}, $\Pi(j)$ is the controller applied to the system at t_j.

Proof: The proof is too long, so omitted, refer to [1],[2]

The theorem manifests that after t_j, the controller is not switched and the real controller is found to make system stable.

Switching controller in this paper has following advantages:

1).Since (11) is the necessary condition that the real observer should meet, it is used as the distinguishing condition of the feasible controllers: If a observer does not meet (11), its corresponding controller must not be the real controller, so the feasible controller set contains the real controller. Since the feasible controller set $F(t_k)$ at t_k is determined in $F(t_{k-1})$ by algorithm 3.1, the relationship:

$$F(t_1) \supseteq F(t_2) \supseteq \ldots\ldots F(t_{k-1}) \supseteq F(t_k) \supseteq \ldots\ldots$$

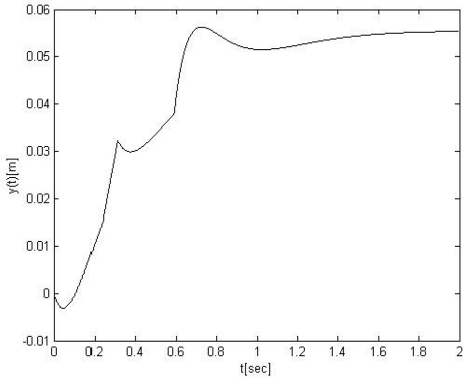

Figure2.The output response of system with feasible controller set method

holds, the scope of the feasible controller set in which the real controller exists can be reduced after the controller is switched each time, i.e. the number of the controllers and observers in Fig1 can be reduced after each switching, so controller can converge fast and online estimation computation burden can be reduced.

2).The switching control method in this paper is very simple and the distinguishing condition is the function of the switching number, it is therefore realistic to implement from a practical point of view.

3).Although the switching control algorithm is designed for continuous system. It (and theorem 4.1) can be generalized to discrete system routinely.

V SIMULATION RESULTS

In this section, the switching control for the following family of 6 plant models is considered. For simplicity and no loss of generation, 6 models adopted for simulation is the certain models. Due to space limitation of the paper, 6 controllers of the models P_i (i=1,2,…6)and 6 observers of the closed-loop systems{P_i, Π_i }, i=1,2,…6, are not given. Each model P_i has the form of

$$P_i : \begin{cases} \dot{x} = A_i x + B_{i1} u + B_{i2}\omega \\ y = C_{i1} x \\ z = C_{i2} x \end{cases}$$

Where

A_1=[3,-17;-67,-3]; B_{11}=[1;-4.3]; B_{12}=[0.1;0.1];
C_{11}=[5,7];C_{12}=C_{11};
A_2=[-75,7.25;-2.25,-8.25];B_{21}=[0.1;0.4]; B_{22}=[0.1;0.1];
C_{21}=[-1.2,4.1];C_{22}=C_{21};
A_3=[14,21.1;-46.5,-25.4];B_{31}=[-0.18;0.33]; B_{32}=[0.1;0.1];
C_{31}=[-0.45,7];C_{32}=C_{31};
A_4=[2.8,-18;-50,-2.4]; B_{41}=[1;-3.3];B_{42}=[0.1;0.1];
C_{41}=[4.5,7];C_{42}=C_{41};
A_5=[3.4,-24;-64,-3]; B_{51}=[1;-4];B_{52}=[0.2;0.1];
C_{51}=[0.5,3];C_{52}=C_{51};
A_6=[-1.9,10.9;-1.3,-12.2];B_{61}=[0;0.1];B_{62}=[0.1;0.1];
C_{61}=[0.2,-0.5];C_{62}=C_{61};

The bounding function f(k) =exp(k)/200, b_0= 0.015, b_1=1.08, $\omega(t) = 5$. Assume that P_6 is the real plant.

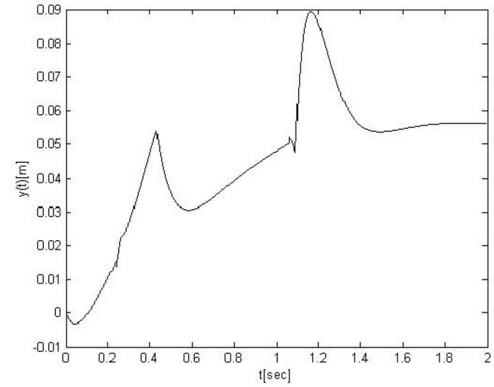

Figure3.The output responds of switching system that does not adopt feasible controller set method (no step 4 in algorithm 3.1)

The output responds of the switching system by algorithm 3.1(feasible controller set method) is shown in Fig2, the output responds of the switching system that does not adopted the feasible controller set method (no step 4 in algorithm 3.1) is shown in Fig3. Compared with the transient response in Fig 3, the transient response in Fig 2 is better than that in Fig 3, which manifests that the feasible controller set method can improve the transient response of the switching system.

For further comparison, Fig 4 and Fig5 are the controller switching instants of algorithm 3.1 and the algorithm that does not adopted the feasible controller set method (no step 4 in algorithm 3.1) respectively. In Fig 4, at the first switching time, the observer O_2 and controller Π_2 and Π_5 must not be the real controller and are excluded from the feasible controller set, Therefore the scope of the feasible controller set is reduced so that the controller converge more rapidly in Fig4 than in Fig 5 and the online estimation computation burden is reduced.

VI CONCLUSIONS

In this paper, a new class of adaptive switching mechanisms is presented. It is assumed that the uncertain plant to be controlled may be described by a model contained in a finite set of linear continuous parameter uncertain system models P, and assumed that a finite set of robust controllers{ Π_i }, i=1,2,……s, has been found so that for each plant $P_i \in P$, i=1,2,……s, there exists one controller Π_i , i=1,2,……s, making the resultant closed-loop system stable, a switching control algorithm is proposed in this paper to select real controller out of controller set { Π_i } with the property of stability of system. This approach is based on the feasible controller set which incorporates simultaneous falsification of a number of controllers, therefore, improves controller converge rate and reduces computation burden. The potential advantages of this approach include the finite convergence for switching, the simplicity of the stability analysis and the applicability to both continuous and discrete system.

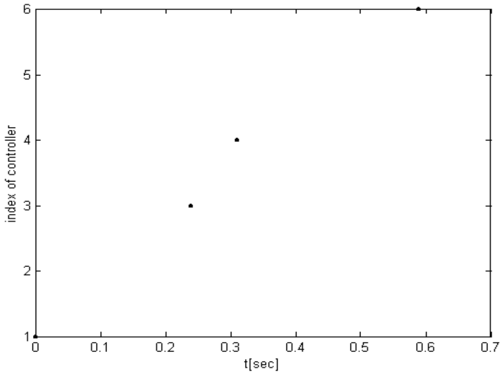

Figure 4 The controller switching instants of algorithm 3.1

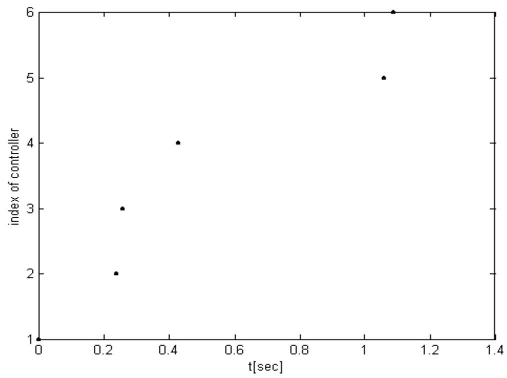

Figure 5 The controller switching instants that does not adapt feasible controller set method (no step4 in algorithm 3.1)

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