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HYDRAULIC MODELING OF SOIL ERORION

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ABSTRACT: The prediction and estimate of soil erosion is fundamental important for understanding the effect of the spatial heterogeneity of underlying surfaces and preventing ecological environment deterioration. This paper summarized soil erosion into three basic dynamics processes, including the process of runoff generation caused by rainfall, the process of sediment yield on hillslopes by overland flow, and the process of runoff concentration and sediment transport on watersheds. Then, a process-based soil erosion model was developed according to the characteristics of erosion on the Loess Plateau. Applying the proposed model, the characteristics of runoff generation and soil erosion on hillslopes were analyzed. In final, the runoff and sediment yield processes in a typical catchment on the loess plateau was estimated, which exhibited a good agreement between predicted results and observation.

1. INTRODUCTION

The Loess Plateau located in the central China with an area of 430,000 km^2, is noted for its ancient culture and serious water and soil loss. About 287,600km^2 of area in this region has an annual erosion rate greater than 1000t/km^2. Moreover, the eroded soil totally amounts to more than 2.2×10^9 tons, among which about 1.6×10^9 tons, on average, is delivered into the Yellow River, rendering the river one of the highest sediment-laden rivers in the world. The serious soil erosion of long period leads to lean soil, desert land formation, arable land diminish and production reduction. Therefore, it is an urgent and challenging task to prevent ecological environment deterioration in western China due to soil erosion.

During the recent decades, there have been extensive investigations on soil erosion in the Loess Plateau area. Many empirical and semi-empirical relationships have been established to estimate runoff and soil erosion quantitatively. However, most of the previous researches aiming at a certain specific erosion type or region only covered very limited parameter range. Since soil and rainfall characteristics substantially vary in different regions, the empirical models do not reflect the overall effect of various factors. Accordingly, there seems to be a shift in emphasis from the empirical approach to the process-based dynamic approach to soil erosion such as WEPP in USA and ANSWERS in Europe. In recent years, some process-based models are also developed in China (Tang and Chen, 1997; Liu et al., 2006). However comparatively, the dynamic study of erosion process on the Loess Plateau is relatively weak. More and more investigations are concerned with the physical mechanism of soil erosion on the Loess Plateau. Therefore, the objective of the current research is to develop the process-based model of soil erosion for examining the fundamental laws of runoff generation and soil erosion, which will certainly be helpful for gaining an insight into these processes on the Loess Plateau of China.

2. BASIC PROCESSES OF SOIL EROSION

Wind and water are the main agents of soil erosion, of which only the latter is examined here. The water erosion is mainly caused by natural rainfall, and is such a process that sheet flow generated during rainfall scours the soil surface and transports the eroded soil into rivers. In general, the erosion process can be divided into three stages (Li et al., 2003): Firstly, when the rainfall intensity is greater than the soil infiltration rate and the surface ponding capability, the excess rain flows down the hillslope under the action of gravity and thus forms the surface sheet flow. Then, when the scouring ability of overland flow is greater than the erosion-resisting capacity of soil, the scour of soil particles is initiated. And finally, the scoured soil is transported downstream by runoff. Therefore, we can simply generalize the soil erosion into three basic dynamics processes, including the process of runoff generation caused by rainfall, the process of sediment yield on hillslopes by overland flow, and the process of runoff concentration and sediment transport on watersheds. Obviously, precipitation, infiltration, overland flow formation and eroding action of runoff on soil are the most fundamental physical processes in soil erosion.

Commonly, overland flow first generates sheet erosion (interrill erosion) and then with increasing flux it causes rill erosion. When a rainstorm occurs, both are commonly observed to coexist on slopes, especially in the Loess Plateau area of China. Investigations showed that rill erosion leads to significant increase in the rates of erosion from hillslopes. In the Loess Plateau area of China, the soil loss from rill erosion, on average, takes up about 70~96% of the total soil loss on hillslopes (Zheng & Kang 1989). In addition, when rill erosion occurs on slopes, most of the sediment eroded by sheet flow only travels a short distance, then, converges into micro-scale channels called rills and is mainly transported by rill flow. This suggests that soil erosion on hillslopes need to be divided into interrill erosion by sheet flow, and rill erosion by concentrated flow in rills.

3. MODEL DESCRIPTION

Soil erosion is a complex process that entails the processes of overland flow, erosion, and runoff concentration and sediment transport. Thus, the proposed model includes three component models: the hillslope rainfall-runoff sub-model, the hillslope soil erosion sub-model and the runoff concentration and sediment transport on watersheds sub-model.

3.1 Rainfall-Runoff Sub-model

On semi-arid hillslopes, all surface runoff occurs in the form of overland flow which is the result of interaction between rainfall and infiltration. When the rainfall intensity exceeds the soil infiltration rate and the capacity of ponding on slope, the water begins to flow down and forms sheet flow. The kinematic wave approximation has long been applied successfully to hillslopes. Under the assumption that rainfall occurs vertically, the steep hillslope gradient for receiving rainfall was significant, and therefore the rainfall intensity for steep slope surfaces need to be modified as $p\cos\theta$ (p is the actual rainfall intensity, θ is the inclination angle of slope). The governing equations of overland flow on the steep hillslopes can be written as:

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=p\cos\theta-i \tag{1}$$

$$q=n^{-1}h^{5/3}S_0^{1/2} \tag{2}$$

where x (m) is the distance down slope, h (m) is the water depth, q (m^{2}s^{-1}) is the unit width discharge, i (ms^{-1}) is the soil infiltration, n (s m$^{-1/3}$) is the Manning roughness coefficient, and S_0 is the slope gradient ($S_0=\sin\theta$).

For runoff formation, soil infiltration is a precursory process. Generally, its influencing factors include the saturate conductivity of soil, initial water content, saturate volumetric water content, cumulative infiltration quantity, soil properties, and etc. A revised Green-Ampt infiltration model was employed to describe the process of rainfall infiltration (Mein and Larson, 1973). When the infiltration rate i, equals the rainfall rate p, the cumulative infiltration, I_p, is expressed as:

$$I_p=(\theta_s-\theta_i)S/(p/K-1) \tag{3}$$

where K (m s^{-1}) is the saturated hydraulic conductivity of the soil, θ_s(m^3 m^{-3}) is the saturated volumetric water content, i.e., the effective porosity, θ_i (m^3 m^{-3}) is the initial volumetric water content, and S (m) is the soil suction. The time to ponding, t_p (s), is obtained from equation (1), $t_p=I_p/p$.

Thus, the infiltration rate during the whole overland flow process can be expressed as a function of cumulative infiltration, I, in the form:

$$\begin{aligned} i&=p, && t\le t_p \\ i&=K[1+(\theta_s-\theta_i)S/I, && t>t_p \end{aligned} \tag{4}$$

In order to express infiltration as a function of time, equation (1) is rewritten as

$$K(t-t_p)=I-I_p-S(\theta_s-\theta_i)\ln\left[\frac{I+S(\theta_s-\theta_i)}{I_p+S(\theta_s-\theta_i)}\right] \tag{5}$$

By the above equations, the real runoff yield process on hillslopes can be simulated reasonably.

3.2 Soil Erosion Sub-model

Interrill erosion and rill erosion are two basic types of soil erosion on rural hillslopes. Therefore, the soil erosion sub-model includes these two parts: interrill erosion and rill erosion, which can be described by the following sediment continuity equation (Liu, et al. 2006):

$$\frac{\partial hC}{\partial t}+\frac{\partial qC}{\partial x}=D_r+D_i \tag{6}$$

where C(kg m^{-3}) is the sediment concentration, D_r (kgs^{-1}m^{-2}) is the rill erosion rate, and D_i (kgs^{-1}m^{-2}) is the interrill erosion rate. Interrill erosion, D_i, in the model represents the process of sediment detachment and delivery to rills.

Interrill erosion

Interrill erosion depends on soil and slope characteristics, vegetation and land use, rainfall intensity, and hydraulic factors of runoff, and reflects the capacity of thin shallow flow to disperse and transport soil on the hillslope. Based on the experiments conducted on a soil flume with artificial rainfall, a formula for interrill erosion rate was derived by Liu et al. (2005):

$$\frac{D_i d}{R_c} = 1.8\times10^{-9}\left(\frac{h}{d}\right)^{1.5}(1.05-0.85e^{-4\sin\theta}) \tag{7}$$

where R_c (kgs^{-1}m^{-1}) is the saturated sediment-transport capacity of interrill flow, which R_c can be calculated according to Low (1989) , and d (m) is the diameter of soil particles.

Rill erosion

Rill flow is different from sheet runoff in hydraulic properties. In general, the process of erosion in rill flow is similar to the process of sediment transport in open channel flow. Due to the limited length of slopes, the sediment transport in rill flow generally belongs to non-equilibrium sediment transport. Assuming that the rill erosion rate is proportional to the difference between the maximum sediment transport capacity of rill flow, T_c (Kgs^{-1}m^{-1}) and the actual sediment transport rate q_s (kgs^{-1}m^{-1}), one has:

$$D_r = \alpha(T_c - q_s) \tag{8}$$

This means that the erosion capacity of flow has a limited value that does not exceed T_c. If the hydraulic condition of flow remains unchanged, the erosion capacity of flow will gradually decrease with increasing sediment concentration of flow until it reaches its maximum value. The maximum sediment transport capacity of rill flow T_c can be calculated by Yalin's formula (Yalin, 1963).

In the above equation, α (m^{-1}) is a coefficient. The reciprocal, $1/\alpha$, has a length dimension and denotes the distance over which the sediment concentration of rill flow reestablishes from zero to the maximum capacity. Thus, parameter α can be named as the restoration coefficient of sediment transport capacity. In general, parameter α is not a constant and depends on flow properties and sediment condition.

Considering the influence of various factors, such as the effective shear stress of the rill flow, the hydraulic radius, the flow velocity, the diameter of soil particles, the density of soil particles and the slope gradient, we obtained the following relationship by using a series of small-scale erosion experiments in laboratory (Liu et al. 2007):

$$\frac{1/\alpha}{R} = 1.5\times10^{4}\left[\frac{\tau-\tau_c}{(\rho_s-\rho)gd}\right]^{0.15}\left(\frac{u}{\sqrt{gd}}\right)^{-1}S_0^{1.5} \tag{9}$$

where R (m) is the hydraulic radius, S_0 is the slope gradient ($S_0 = \sin\theta$), τ (Pa) is the flow shear stress, τ_c (Pa) is the critical shear stress.

3.2 Runoff Concentration and Sediment Transport Sub-model

In practical cases, the surface landform of a watershed is very complex, especially in the loess plateau area, and the runoff commonly concentrates from slope in to channels because of the effect of irregular topography. Scoging (1992) firstly studied the process of runoff converging on hillslopes, and developed a distributed model of overland flow. Based on the previous experience, a two-dimensional kinematic wave model, which can adequately reflect the runoff concentration process, was developed (Liu et al. 2004). The 2-D form of kinematic wave theory can be expressed as follows.

$$\begin{cases} \dfrac{\partial h}{\partial t}+\dfrac{\partial q_x}{\partial x}+\dfrac{\partial q_y}{\partial y} = p\cos\theta - i, \\ q = uh = n^{-1}h^{5/3}S_{0x}^{1/2},\ q_x = q\cos\gamma,\ q_y = q\sin\gamma. \end{cases} \tag{10}$$

where q is the unit width discharge of runoff in the flow direction; u is the runoff velocity in the flow direction, q_x and q_y are the unit width discharge components in the x and y directions, respectively; and γ is the angle between the flow direction and the x coordinate.

From Eq.(10), it is obvious that the expressions originate from the 1-D formulation of the kinematic wave theory. Then, the runoff discharge can be divided into the x and y directions according to the flow direction. A different flowline routine is specified in every grid and the inflow and the outflow discharge of the adjacent grids are calculated. The average slope can be used to express the angle of calculation element. And the flow vector direction angle γ for a calculated element can be calculated by following formula:

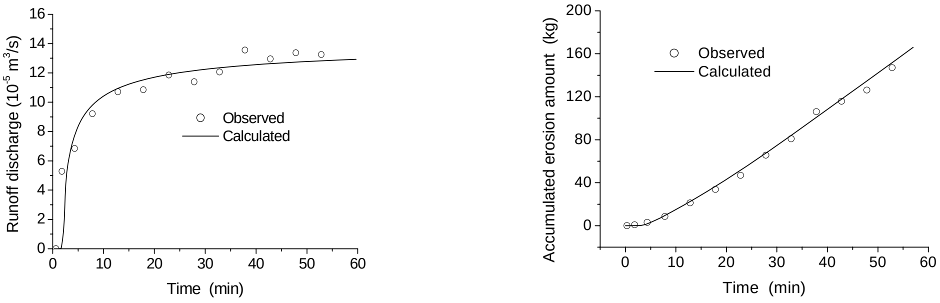
$$\mathrm{tg}\gamma = \frac{\mathrm{d}x}{\mathrm{d}y}\left[\frac{z_A + z_C - z_B - z_D}{z_A + z_B - z_C - z_D}\right]. \tag{11}$$

in which, z_A, z_B, z_C, z_D are the heights of four corner points, respectively; the dx and dy values are the projections on the horizontal plane of the grid step lengths in the x and y directions, respectively.

The stream network of the whole catchment can be ascertained according to the conflux cells and the observed stream network. The flow in stream network is simulated by 1-D diffusion wave model

4. MODEL VALIDATION

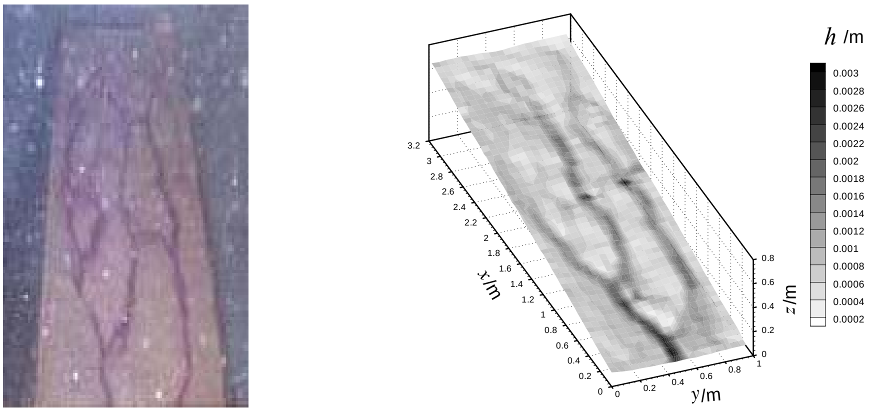
A data set obtained from experiments conducted on a soil flume with artificial rainfall was used to verify the overland flow generation and soil erosion sub-models on hillslopes (Liu et al. 2006). The soil is the loess soil with a median grain size of 0.02mm. The bulk density of soil is 1.33g/cm^3, the initial moisture content of soil is 0.2206, and the porosity of the soil is 0.5027. The infiltration coefficient and the suction of loess soil are $1.67\times10^{-6} m/s$ and 0.15m, respectively. Comparisons of one case of observed values and the values predicted by the proposed model are shown in Fig. 1 (The rainfall intensity: 2.06 mm/min, the inclination angle of slope: 15^0). Overall, good agreements were found between observed and predicted values of discharge and accumulated erosion amounts.



(a) Total runoff discharge (b) Accumulated erosion amount

Fig. 1 Comparison of experimental and predicted results

Another experiment was carried out to study the process of flow concentration on a hillslope with an up-and-down landform. The test plot was a 320 cm long, 100 cm wide wooden-flume. The slope gradient of the soil flume was secured at 10° during experiment. The rainfall intensity adopted in this experiment was 1.6mm/min. Fig. 2 shows the flow visualization result of concentrated flow in experiment and the simulated results of runoff depth distribution. The agreement between simulation results and experimental observations is rather good.



(a) Flow visualization results of concentrated flow (b) Simulated results of runoff depth distribution

Fig. 2. Comparison of the observed and predicted concentrated flow route

5. CHARACTERISTICS OF RUNOFF GENERATION AND EROSION ON HILLSLOP

Using the proposed model, we analyzed the characteristics of runoff generation and soil erosion on a hillslope (Li et al. 2003; Liu et al. 2006). Especially, the slope length and gradient play important roles in the processes of runoff generation and erosion on hillslopes. Fig. 3 showed that the slope gradient plays both positive and negative roles on the flow velocity and shear stress of overland flow. The flow velocity and shear stress increases at first and then began to decrease when the slope gradient reaches their respective critical values. Although the corresponding critical slopes are not equal, both of them are estimated within the range of about 40^{O}~50^{O}. Fig. 4 showed the effect of the slope length and gradient on soil erosion on a hillslope. The calculated results show that the erosion amount remarkably and nonlinearly increased as the slope length increased, which reveals that short slopes would be beneficial for decreasing soil erosion. In addition, the effect of the slope gradient on soil erosion can be both positive and negative. Thus, there exists a critical slope gradient for soil erosion, which is about 25^{o} for the accumulated erosion amount.

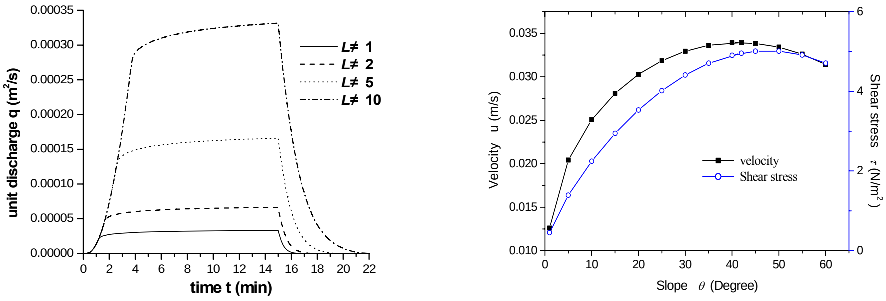


Fig. 3. Variation of runoff generation with slope length and slope degree.

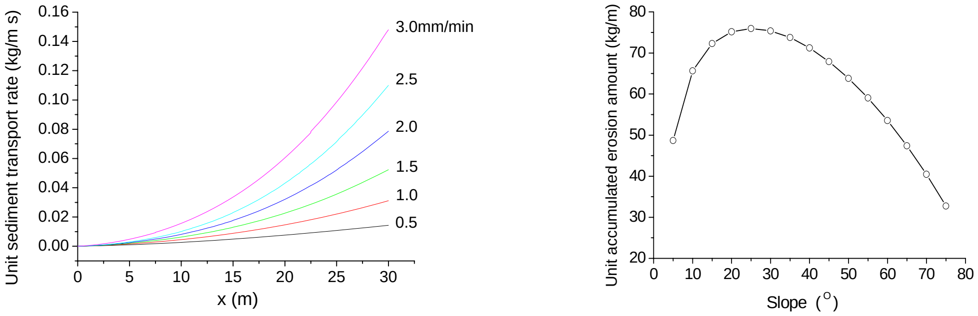


Fig. 4. Influence of slope length and slope gradient on soil erosion

6. APPLICATION OF MODEL IN A SMALL CATCHMENT

A small catchment, the Heicaohe catchment located in the Loess Plateau area in Yellow River basin was chosen to test the performance of the proposed model. The catchment area is 23.7 km^2, and the main soil type is the silt loam. The hydrological station of Datianbao is situated at the main stream near the outlet of the catchment. The topography map of the Heicaohe catchment is shown in Fig. 5.

Using the proposed model, we simulated the process of runoff discharge and sediment concentration at hydrological station of Datianbao of the Heicaohe catchment, for September 15th, 1985 storm event. The comparisons between observed and simulated results are shown in Fig.6. Overall, good agreements were found between observed and predicted values for the rainfall-runoff yield and the sediment yield in the catchment. It also demonstrated that the proposed model is capable of adequately simulating the process of runoff yield and soil erosion on small watersheds.

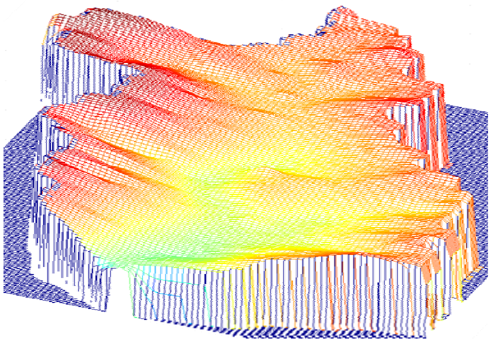


Fig. 5 Topography map of Heicaohe catchment

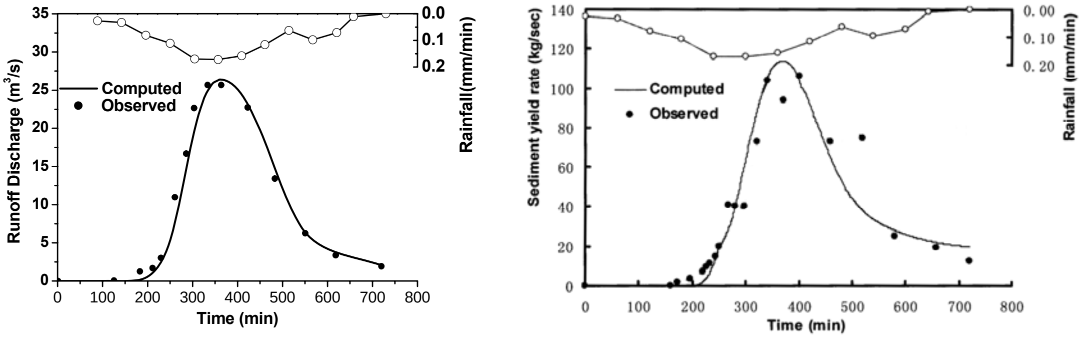


Fig. 6 Comparisons between observed and simulated runoff discharge and sediment yield rate at the Datianbao station for the storm event in September 15th, 1985.

7. CONCLUDING REMARKS

The Loess Plateau of China is well known for its unique landform and severe soil erosion. Many empirical and semi-empirical relationships have been established to estimate runoff and soil erosion quantitatively. However, most of the previous researches aiming at a certain specific erosion type or region only covered very limited parameter range. Accordingly, the research of soil erosion at present tends to shift from the empirical approach to the process-based dynamic description.

The soil erosion can be simply divided into three basic dynamics processes, including the process of runoff generation caused by rainfall, the process of sediment yield on hillslopes by overland flow, and the process of runoff concentration and sediment transport on watersheds.

A process-based soil erosion model was developed according to the characteristics of soil erosion on the Loess Plateau. The model includes three component models: the rainfall-runoff sub-model on hillslopes, the soil erosion sub-model on hillslopes and the runoff concentration and sediment transport sub-model on watersheds.

Some basic characteristics of runoff generation and soil erosion on hillslopes were analyzed by simulation. Moreover, the influential factors of soil erosion can be used as valuable references in reducing erosion as an effective measure of erosion prevention.

Applying the proposed model to a typical catchment in the loess plateau area, the runoff and sediment yield process was estimated, which exhibited a good agreement between predicted results and observation.

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