

Research Paper

Electro-osmotic consolidation of soil with variable compressibility, hydraulic conductivity and electro-osmosis conductivity



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ABSTRACT

In present study, the non-linear variations of soil compressibility, hydraulic and electro-osmosis conductivities were analyzed through laboratory experiments, and incorporated in a one-dimensional model. The analytical solutions for excess pore water pressure and degree of consolidation were derived, and numerical simulations were performed to verify its effectiveness. The results indicated that both the non-linear variations of hydraulic and electro-osmosis conductivities showed remarkable impacts on the excess pore water pressure and degree of consolidation, especially for soils with relative high compressibility. A further comparison with previous analytical solutions indicated that more accurate predictions could be obtained with the proposed analytical solutions.

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1. Introduction

Over the past few decades, there has been a substantial development of infrastructure on soft foundations worldwide. Various treatment methods have been proposed for the improvement of these soft foundations, among which electro-osmotic consolidation has proven to be a promising method, especially for soils with low permeability [1–10]. Unlike the traditional methods such as surcharge and vacuum preloading which dewater soil mass by applying a hydraulic gradient, electro-osmotic consolidation involves pairs of anodes and cathodes installed in the soil mass, through which an electrical field is applied and pore water is driven from the anode to cathode under the electrical gradient. Similar to Darcy's law, the velocity of pore water flow v_e caused by the electrical gradient can be expressed as $k_e \times i_e$, where i_e means electrical gradient and k_e means electro-osmosis conductivity that describes the velocity of pore water under a unit electrical gradient. For different soils, the hydraulic conductivity k_h may change from about 1×10^{-8} cm/s in clay to about 1×10^{-4} cm/s in sand, while k_e is generally in the range of 1×10^{-5} to 1×10^{-4} cm²/ (Vs). As a result, a small electrical gradient can balance flows caused by large hydraulic gradient in soft soil with low permeabil-

ity, and electro-osmosis can be much more efficient than the traditional techniques for soft soil improvement [5, 11–13].

Based on the assumption that the pore water flow resulted from hydraulic gradient and electrical gradient can be linearly superimposed, the governing equation for electro-osmotic consolidation was developed and many analytical solutions were derived based on different conditions to analyze the development of pore water pressure [14–20]. Esrig [14] developed a one-dimensional (1D) model for electro-osmotic consolidation and obtained the analytical solutions for pore water pressure and degree of consolidation considering a permeable cathode and an impermeable anode. Wan and Mitchell [15] further coupled electro-osmotic consolidation with surcharge preloading in a 1D model. Shang [16] and Xu et al. [21] proposed a 2D model in vertical plane to account for the combined action of electro-osmosis with surcharge preloading and vacuum preloading. Su and Wang [22] presented a 2D model in horizontal plane and derived the analytical solutions under different boundary conditions. Li et al. [17] analyzed the average pore water pressure in soils submitted to an axisymmetric electrical field. Wu and Hu [19] developed an axisymmetric model with coupled horizontal and vertical seepage and derived the analytical solution without the equal strain hypothesis. These mathematical analyses have generated significant knowledge pertaining to electro-osmotic consolidation and provided useful formulas for engineering design. However, the electrical and mechanical properties of soil are assumed constant during the derivation of these

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Nomenclature	
a, a_0	the coefficient of compressibility and the initial coefficient of compressibility
b	calculating factor related to C_c , M and N
A	sectional area of the soil sample
C_c	compression index
C_{v0}	the initial coefficient of consolidation
C_0	calculating factor for ultimate excess pore water pressure
e, e_0	void ratio and initial void ratio
G_n	calculating factors
H	height of the analytical model
I, J	calculating factors
k_e, k_{e0}	electro-osmosis conductivity and initial electro-osmosis conductivity
k_h, k_{h0}	hydraulic conductivity and initial hydraulic conductivity
L	length of the soil sample
M, N	factors describing the change in hydraulic and electro-osmosis conductivities resulted from the change in void ratio
p_0	surcharge preloading
q	volume of the discharged water due to electro-osmosis
Q	defined variable related to the excess pore water pressure
Q_{ult}	defined variable related to ultimate excess pore water pressure
t	time period
T_v, T_c, T_e	time factors
u	excess pore water pressure
u_{ult}	ultimate excess pore water pressure
U	degree of consolidation
v_z	the pore water flow in the vertical direction
V	voltage
V_0	the applied voltage in the electro-osmosis test
W	ratio between excess pore water pressure and surcharge preloading
W_{ult}	ratio between ultimate excess pore water pressure and surcharge preloading
Z	ratio between vertical position and height of the model
α, θ	calculating factors
β	calculating factor related to surcharge preloading and initial effective stress
γ_w	unit weights of water and saturated soil
σ', σ_0	effective stress and initial effective stress
ε	the weighted factor
ξ_n	solution for $\tan(\xi/2) = \xi/\theta$, where n equals 1,2,3.....
λ_n	equals $(\xi_n^2 + \theta^2)/4$

analytical solutions. In fact, the flow of pore water from anode to cathode during electro-osmosis causes the decrease in water content and void ratio of the treated soil, and leads to non-linear variations in soil properties such as compressibility, hydraulic conductivity and electro-osmosis conductivity [1,4,5,7,13,23–28]. Such variations would inevitably affect the development of pore water pressure during electro-osmotic consolidation, and the predictions from the existing analytical solutions with constant soil properties would be inaccurate.

Although being ignored in the analytical solutions for electro-osmotic consolidation, the non-linear variations of soil compressibility and hydraulic conductivity have already been investigated in many consolidation theories [29–38]. Davis and Raymond [29] derived the analytical solution for pore water pressure with the assumption of non-linear compressibility and constant coefficient of consolidation. Poskitt [31] further coupled the relationships between void ratio (e) and effective stress (σ'), hydraulic conductivity (k_h) into a vertical consolidation model. Lekha et al. [33] presented closed form analytical solutions for the pore water pressure and degree of consolidation for the particular cases of $e - \log(\sigma')$ and $e - \log(k_h)$ responses. In these studies, the relationships of $e - \log(\sigma')$ and $e - \log(k_h)$ were developed and incorporated to account for the non-linear variations of soil compressibility and permeability.

Compared to the traditional consolidation problem, electro-osmotic consolidation involves not only the non-linear variations of soil compressibility and permeability but also the change in electro-osmosis conductivity. In this study, a series of experiments were performed to investigate the variations of hydraulic and electro-osmosis conductivities during electro-osmotic consolidation. Afterwards, the relationships between the hydraulic conductivity, electro-osmosis conductivity and void ratio were developed based on the experiment results, and further incorporated into a 1D model for electro-osmotic consolidation together with the conventional $e - \log(\sigma')$ response. The analytical solutions for excess pore water pressure and degree of consolidation were derived

and compared with that from Wan and Mitchell (1976) to investigate the effects of the non-linear variations of soil properties.

2. Experimental study

A kaolinite from Jiangsu Province, China was used to conduct permeability and electro-osmosis tests. The basic properties and chemical composition of the kaolinite were listed in Table 1. The as-received kaolinite was first oven dried, then mixed with water at a water content of 10% and compacted into the test devices in five layers according to the pre-determined void ratio in the range of 1.373–0.919, and finally saturated under a vacuum.

The hydraulic conductivities of the kaolinite samples were monitored with the falling head permeability test, and the electro-osmosis conductivities were measured using a self-designed apparatus 90 mm in diameter and 400 mm in height as shown in Fig. 1. The anode platen was placed on the bottom of the kaolinite sample, while the porous cathode platen on the top, allowing the drainage of pore water into a graduated cylinder. In order to eliminate the effect of hydraulic gradient, the bottom of the kaolinite sample was connected to a water reservoir with a

Table 1
Geotechnical properties and chemical composition of the kaolinite.

Geotechnical property	Value
Initial water content, w_0 (%)	1.31
Liquid limit, w_L (%)	58
Plastic limit, w_p (%)	23
Plasticity index, I_p (%)	35
Specific gravity, G_s	2.61
Chemical composition	2.0
SiO ₂	50
Al ₂ O ₃	35
Fe ₂ O ₃	3.2
K ₂ O + Na ₂ O	4.2

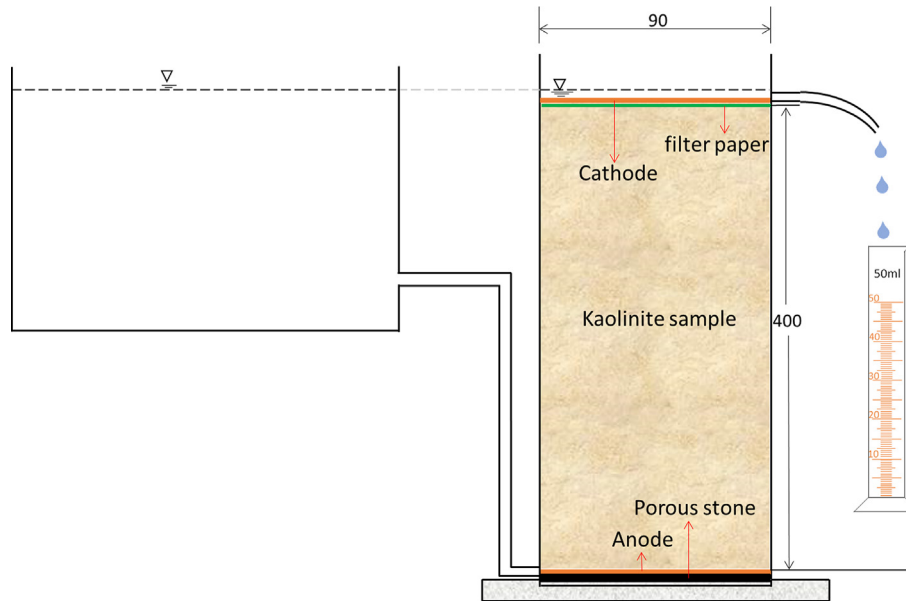


Fig. 1. One-dimensional apparatus for electro-osmosis experiment (length unit: mm).

water level as high as the top surface of the kaolinite sample. After the saturation of the kaolinite sample and the connection to the water blank, the whole system was allowed to stand for at least 2 days to reach equilibrium. Afterwards, an electrical field was applied and the electro-osmosis conductivity was calculated with the following equation,

$$k_e = \frac{q \cdot L}{A \cdot t \cdot V_0} \quad (1)$$

where q is the volume of the discharged water due to electro-osmosis; L and A are the length and cross sectional area of the soil sample; t is time period; V_0 is the applied voltage.

Fig. 2 shows the hydraulic and electro-osmosis conductivities of the kaolinite. With the decrease in void ratio, both k_h and k_e decrease, and their relationships with void ratio can be expressed as following,

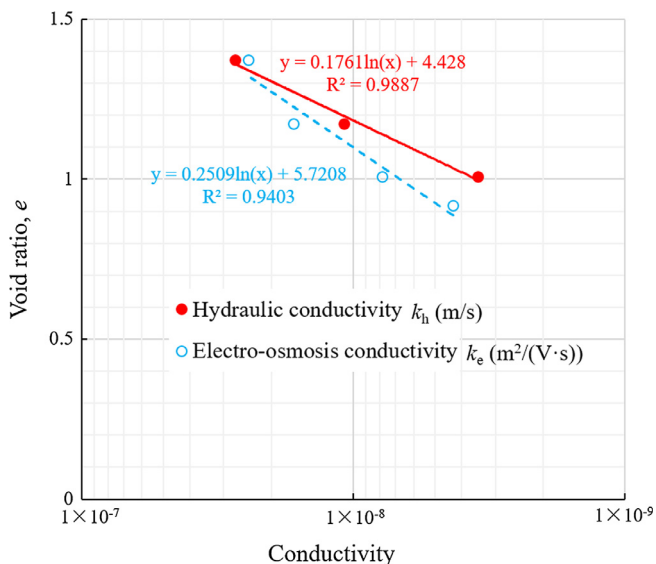


Fig. 2. Relationships between void ratio and hydraulic conductivity and electro-osmosis conductivity.

$$e = 4.428 + 0.1761 \ln(k_h) = 1.373 + 0.4055 \times \log[k_h / (2.92 \times 10^{-8} \text{ m/s})] \quad (2)$$

$$e = 5.7208 + 0.2509 \ln(k_e) = 1.373 + 0.5777 \times \log[k_e / (2.98 \times 10^{-8} \text{ m}^2 / (\text{V s}))] \quad (3)$$

Previous studies have investigated the variation of void ratio during the consolidation process [29–31,33], and the results indicated that the relationship between void ratio and effective stress could be written as,

$$e = e_0 - C_c \cdot \log(\sigma' / \sigma_0) \quad (4)$$

in which e_0 is initial void ratio, σ_0 is initial effective stress, and C_c is the compression index.

Similar to Eq. (4), the general form for $e - \log(k_h)$ and $e - \log(k_e)$ responses can be obtained according to Eqs. (2) and (3),

$$e = e_0 + M \cdot \log(k_h / k_{h0}) \quad (5)$$

$$e = e_0 + N \cdot \log(k_e / k_{e0}) \quad (6)$$

in which k_{h0} and k_{e0} are initial hydraulic and electro-osmosis conductivities corresponding to e_0 ; M and N are factors that reflect the changes in hydraulic and electro-osmosis conductivities resulted from the change in void ratio.

Eqs. (4)–(6) together describe the non-linear variations of soil compressibility, hydraulic conductivity and electro-osmosis conductivity during the consolidation process.

3. Theoretical analysis

Similar to previous studies, a schematic diagram of the 1D model for electro-osmotic consolidation is developed as shown in Fig. 3, with the anode on the bottom and cathode on the top [1,12–13,18]. The bottom boundary is impermeable and the top boundary is permeable. A surcharge preloading p_0 is applied on the top boundary of the model. The following assumptions are made to develop the analytical model for electro-osmotic consolidation.

- (1) The soil is homogeneous and fully saturated, and the pore water and soil grain are incompressible.

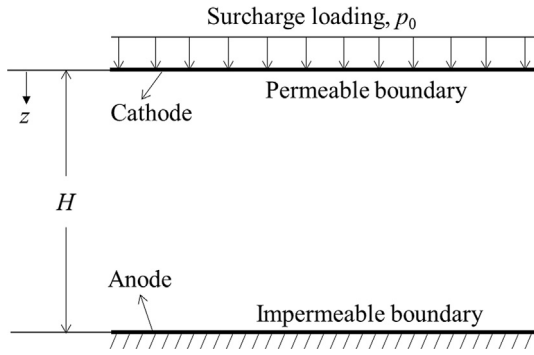


Fig. 3. Diagram of the one-dimensional analytical model for electro-osmotic consolidation.

- (2) Both the drainage of pore water and the compression of soil layer occur in the vertical direction.
- (3) The velocity of pore water flow due to electro-osmosis is directly proportional to the electrical gradient, and can be linearly superimposed with that due to hydraulic gradient.
- (4) The relationships between e and σ' , k_h and k_e in Eqs. (4)–(6) hold.
- (5) The loading is instantaneously applied, and the small strain hypothesis is adopted.
- (6) The pore water flow caused by thermal gradient and chemical concentration gradient is neglected.

The combined pore water flow during electro-osmotic consolidation can be described as following [7, 14, 15, 19],

$$v_z = -\frac{k_h}{\gamma_w} \frac{\partial u}{\partial z} - k_e \frac{\partial V}{\partial z} \quad (7)$$

where v_z is the pore water flow in the vertical direction; γ_w is the unit weight of water; u is excess pore water pressure; V is the voltage.

According to the conservation of pore water in a saturated soil-water system,

$$\frac{\partial v_z}{\partial z} = \frac{-1}{1 + e_0} \frac{\partial e}{\partial t} = \frac{1}{1 + e_0} \left(\frac{\partial e}{\partial \sigma'} \right) \frac{\partial u}{\partial t} \quad (8)$$

Substituting Eq. (7) into Eq. (8), and making use of the substitutions $W = u/p_0$ and $Z = z/H$ (H is the height of the model), the following equation can be obtained,

$$\frac{1}{H^2} \frac{\partial}{\partial Z} \left(\frac{k_h \cdot p_0}{\gamma_w} \frac{\partial W}{\partial Z} + k_e \frac{\partial V}{\partial Z} \right) = \frac{p_0}{1 + e_0} \left(\frac{\partial e}{\partial \sigma'} \right) \frac{-\partial W}{\partial t} = \frac{p_0}{1 + e_0} a \frac{\partial W}{\partial t} \quad (9)$$

where a denotes the coefficient of compressibility. Further simplifying Eq. (9) to,

$$\begin{aligned} \frac{\partial W}{\partial t} &= \frac{k_{h0}(1 + e_0)}{\gamma_w H^2 a} \cdot \frac{\partial}{\partial Z} \left(\frac{k_h}{k_{h0}} \frac{\partial W}{\partial Z} + \frac{k_e \gamma_w}{k_{h0} p_0} \frac{\partial V}{\partial Z} \right) \\ &\Rightarrow \frac{\partial W}{\partial T_v} = \frac{a_0}{a} \cdot \frac{\partial}{\partial Z} \left(\frac{k_h}{k_{h0}} \frac{\partial W}{\partial Z} + \frac{k_e \gamma_w}{k_{h0} p_0} \frac{\partial V}{\partial Z} \right) \end{aligned} \quad (10)$$

where a_0 is the initial coefficient of compressibility; T_v is the time factor and equals to $C_{v0} \cdot t/H^2$; C_{v0} is the initial coefficient of consolidation.

According to Lekha et al. [33], the non-linear variations of soil compressibility, hydraulic conductivity and electro-osmosis conductivity can be rewritten as,

$$\begin{aligned} \frac{a_0}{a} &= \frac{\sigma'}{\sigma_0} = \frac{\sigma_0 + p_0 - W \cdot p_0}{\sigma_0} = \left(1 + \frac{p_0}{\sigma_0} \right) \cdot \left(1 - \frac{W p_0}{1 + \frac{p_0}{\sigma_0}} \right) \\ &= \left(1 + \frac{p_0}{\sigma_0} \right) \cdot (1 - \beta W) \end{aligned} \quad (11)$$

$$\frac{k_h}{k_{h0}} = \left(\frac{\sigma'}{\sigma_0} \right)^{-C_c/M} = \left(\frac{a_0}{a} \right)^{-C_c/M} = \left[\left(1 + \frac{p_0}{\sigma_0} \right) \cdot (1 - \beta W) \right]^{-C_c/M} \quad (12)$$

$$\frac{k_e}{k_{e0}} = \left(\frac{\sigma'}{\sigma_0} \right)^{-C_c/N} = \left(\frac{a_0}{a} \right)^{-C_c/N} = \left[\left(1 + \frac{p_0}{\sigma_0} \right) \cdot (1 - \beta W) \right]^{-C_c/N} \quad (13)$$

where β is defined as $(p_0/\sigma_0)/(1 + p_0/\sigma_0)$.

Substituting Eqs. (11)–(13) into Eq. (10),

$$\begin{aligned} \frac{\partial W}{\partial T_v} &= \left(1 + \frac{p_0}{\sigma_0} \right) (1 - \beta W) \cdot \frac{\partial}{\partial Z} \left\{ \left[\left(1 + \frac{p_0}{\sigma_0} \right) (1 - \beta W) \right]^{-C_c/M} \frac{\partial W}{\partial Z} \right\} \\ &+ \left(1 + \frac{p_0}{\sigma_0} \right) (1 - \beta W) \\ &\cdot \frac{\partial}{\partial Z} \left\{ \left[\left(1 + \frac{p_0}{\sigma_0} \right) (1 - \beta W) \right]^{-C_c/N} \cdot \frac{\partial V}{\partial Z} \cdot \frac{k_{e0} \gamma_w}{k_{h0} p_0} \right\} \end{aligned} \quad (14)$$

Defining a new parameter Q as,

$$\begin{aligned} Q &= (1 - \beta W)^{1+C_c/N-C_c/M} - \beta \cdot \left(1 + \frac{C_c}{N} - \frac{C_c}{M} \right) \cdot \left(1 + \frac{p_0}{\sigma_0} \right)^{C_c/M-C_c/N} \\ &\cdot \frac{k_{e0} \gamma_w}{k_{h0} p_0} \cdot \frac{\partial V}{\partial Z} \cdot Z = (1 - \beta W)^{1+C_c/N-C_c/M} - C_0 \cdot Z \end{aligned} \quad (15)$$

in which $C_0 = \beta \cdot (1 + C_c/N - C_c/M) \cdot (1 + p_0/\sigma_0)^{C_c/M-C_c/N} \cdot k_{e0} \gamma_w / k_{h0} p_0 \cdot \partial V / \partial Z$.

Then Eq. (14) is simplified to,

$$\begin{aligned} \frac{\partial Q}{\partial T_v} &= (Q + C_0 \cdot Z) \cdot \left(1 + \frac{p_0}{\sigma_0} \right)^{1-C_c/M} \cdot \frac{\partial}{\partial Z} \left[(1 - \beta W)^{-C_c/N} \frac{\partial Q}{\partial Z} \right] \\ &\Rightarrow \frac{\partial Q}{\partial T_c} = (Q + C_0 \cdot Z) \cdot \frac{\partial}{\partial Z} \left[(1 - \beta W)^{-C_c/N} \frac{\partial Q}{\partial Z} \right] \\ &= (Q + C_0 \cdot Z) \cdot \frac{\partial}{\partial Z} \left[(Q + C_0 \cdot Z)^b \frac{\partial Q}{\partial Z} \right] \\ &\Rightarrow \frac{\partial Q}{\partial T_c} = (Q + C_0 \cdot Z)^{b+1} \frac{\partial^2 Q}{\partial Z^2} + (Q + C_0 \cdot Z)^b \cdot b \cdot \frac{\partial Q}{\partial Z} \cdot \frac{\partial Q}{\partial Z} \\ &+ (Q + C_0 \cdot Z)^b \cdot b \cdot C_0 \cdot \frac{\partial Q}{\partial Z} \end{aligned} \quad (16)$$

where $T_c = T_v \cdot (1 + p_0/\sigma_0)^{(1-C_c/M)}$; $b = (-C_c/N)/(1 + C_c/N - C_c/M)$.

According to the description of the model (Fig. 3), the boundary and initial conditions of the problem are,

$$\begin{cases} Z = 0, u = 0 \Rightarrow Q = 1 \\ Z = 1, \frac{k_h}{\gamma_w} \frac{\partial u}{\partial z} + k_e \frac{\partial V}{\partial z} = 0 \Rightarrow \frac{\partial Q}{\partial Z} = 0 \\ t = 0, u = p_0 \Rightarrow Q = (1 - \beta)^{1+C_c/N-C_c/M} - C_0 \cdot Z \end{cases} \quad (17)$$

Eq. (16) is non-linear in Q and therefore does not have a general solution with the boundary and initial conditions in Eq. (17). In order to solve Q from Eq. (16), the coefficient terms on the right hand side of Eq. (16) are replaced by their weighted average value. Specifically, when $t = 0$, $Q = (1 + p_0/\sigma_0)^{(C_c/M-C_c/N-1)} - C_0 \cdot Z$; when t tends to infinity, the pore water flow caused by electro-osmosis from the anode to cathode is exactly balanced by that caused by hydraulic gradient from the cathode to anode, and the following equation can be obtained,

$$\frac{k_h}{\gamma_w} \cdot \frac{\partial u}{\partial z} + k_e \cdot \frac{\partial V}{\partial z} = 0 \Rightarrow \frac{\partial W}{\partial Z} + \frac{k_e \cdot \gamma_w}{k_h \cdot p_0} \cdot \frac{\partial V}{\partial Z} = 0 \quad (18)$$

Therefore,

$$\begin{aligned} \frac{\partial Q}{\partial Z} &= -\beta \cdot \left(1 + \frac{C_c}{N} - \frac{C_c}{M} \right) \cdot (1 - \beta W)^{C_c/N-C_c/M} \cdot \frac{\partial W}{\partial Z} - C_0 \\ &= \beta \cdot \left(1 + \frac{C_c}{N} - \frac{C_c}{M} \right) \cdot \left(1 + \frac{p_0}{\sigma_0} \right)^{C_c/M-C_c/N} \cdot \frac{k_{e0} \gamma_w}{k_{h0} p_0} \frac{\partial V}{\partial Z} - C_0 = 0 \end{aligned} \quad (19)$$

Considering the top boundary condition, Q equals 1 when t tends to infinity. A weighted average value for $(Q + C_0 Z)$ is then assumed as following,

$$(Q + C_0 Z)_{av} = \varepsilon \cdot \left(1 + \frac{p_0}{\sigma_0}\right)^{C_c/M - C_c/N - 1} + (1 - \varepsilon) \cdot (1 + 0.5C_0) \quad (20)$$

where ε is the weighted factor. Substituting Eq. (20) into Eq. (16),

$$\frac{\partial Q}{\partial T_e} = \frac{\partial^2 Q}{\partial Z^2} + \alpha \cdot \frac{\partial Q}{\partial Z} \cdot \frac{\partial Q}{\partial Z} + \theta \cdot \frac{\partial Q}{\partial Z}$$

$$T_e = \left[\varepsilon \cdot \left(1 + \frac{p_0}{\sigma_0}\right)^{C_c/M - C_c/N - 1} + (1 - \varepsilon) \cdot (1 + 0.5C_0) \right]^{b+1} \cdot T_c \quad (21)$$

$$\alpha = \frac{b}{\varepsilon \cdot \left(1 + \frac{p_0}{\sigma_0}\right)^{C_c/M - C_c/N - 1} + (1 - \varepsilon) \cdot (1 + 0.5C_0)}, \theta = C_0 \cdot \alpha$$

Combining Eq. (21) with Eq. (17), Q and u can be solved (details are given in Appendix A),

$$Q = \frac{1}{\alpha} \ln \left[\sum_{n=1}^{\infty} G_n e^{-\lambda_n T_e} \cdot e^{-\frac{1}{2}\theta Z} \cdot \sin\left(\frac{1}{2}\xi_n Z\right) + 1 \right] + 1 \quad (22)$$

$$u = \frac{p_0}{\beta} - \frac{p_0}{\beta} \cdot \left\{ \frac{1}{\alpha} \ln \left[\sum_{n=1}^{\infty} G_n e^{-\lambda_n T_e} \cdot e^{-\frac{1}{2}\theta Z} \cdot \sin\left(\frac{1}{2}\xi_n Z\right) + 1 \right] + 1 + C_0 Z \right\}^{1/(1+C_c/N - C_c/M)} \quad (23)$$

where ξ_n is a solution for $\tan(\xi/2) = \xi/\theta$; $\lambda_n = (\xi_n^2 + \theta^2)/4$; G_n can be expressed as,

$$G_n = \frac{I \cdot \frac{2}{\xi_n} \cdot \left(1 - e^J \cdot \frac{\theta}{\sqrt{\theta^2 + \xi_n^2}} + J \cdot e^J \cdot \frac{2}{\xi_n} \cdot \frac{\xi_n}{\sqrt{\theta^2 + \xi_n^2}}\right)}{\left(1 + J^2 \cdot \frac{4}{\xi_n^2}\right) \cdot \left(\frac{1}{2} - \frac{\theta}{\theta^2 + \xi_n^2}\right)}$$

$$- \frac{\frac{2}{\xi_n} \left(1 - e^{\frac{1}{2}\theta} \cdot \frac{\theta}{\sqrt{\theta^2 + \xi_n^2}} + \frac{1}{2} \theta \cdot e^{\frac{1}{2}\theta} \cdot \frac{2}{\xi_n} \cdot \frac{\xi_n}{\sqrt{\theta^2 + \xi_n^2}}\right)}{\left(1 + \frac{\theta^2}{\xi_n^2}\right) \cdot \left(\frac{1}{2} - \frac{\theta}{\theta^2 + \xi_n^2}\right)} \quad (24)$$

$$I = e^{\alpha \left[\left(1 + \frac{p_0}{\sigma_0}\right)^{C_c/M - C_c/N - 1} - 1 \right]}, \quad J = \frac{1}{2}\theta - \alpha C_0$$

The degree of consolidation U can be calculated as,

$$U = \frac{\int_0^H (p_0 - u) dz}{\int_0^H (p_0 - u_{t \rightarrow \infty}) dz} = \frac{(p_0 - \frac{p_0}{\beta})H + \int_0^H \frac{p_0}{\beta} \left\{ \frac{1}{\alpha} \ln \left[\sum_{n=1}^{\infty} G_n e^{-\lambda_n T_e} \cdot e^{-\frac{1}{2}\theta Z} \cdot \sin\left(\frac{1}{2}\xi_n Z\right) + 1 \right] + 1 + C_0 Z \right\}^{\frac{1}{1 + \frac{C_c}{N} - \frac{C_c}{M}}} dz}{(p_0 - \frac{p_0}{\beta})H + \frac{p_0 H}{\beta C_0 \left(\frac{1}{1 + \frac{C_c}{N} - \frac{C_c}{M}} + 1 \right)} \cdot \left[(1 + C_0)^{\frac{1}{1 + \frac{C_c}{N} - \frac{C_c}{M}} + 1} - 1 \right]} \quad (25)$$

The integral in Eq. (25) is highly related to the value of $1/(1 + C_c/N - C_c/M)$ and cannot be expressed by elementary functions. Therefore, the Newton–Cotes formula is used to estimate its value.

4. Discussion

Due to the assumption about the coefficient terms in Eq. (16), the solution in Eq. (23) for the excess pore water pressure is actually not accurate. In order to examine the effectiveness of the solution, a numerical model is developed based on the governing equation in Eq. (10), the boundary and initial conditions in

Table 2
Basic parameters for electro-osmotic consolidation in the analytical model.

Parameter	Value
Unit weight of water, γ_{sat} (kN/m ³)	10
Initial hydraulic conductivity, k_{h0} (m/s)	2×10^{-8}
Initial electro-osmosis conductivity, k_{eo} (m ² s ⁻¹ V ⁻¹)	2×10^{-9}
Surcharge preloading, p_0 (kPa)	50
Initial effective stress, σ_0 (kPa)	10
Initial void ratio, e_0	2.0
Initial coefficient of compressibility, a_0 (MPa ⁻¹)	8.7
Compressibility index, C_c	0.2
M	2
N	8
Applied voltage, V_0 (V)	40
Model height, H (m)	1

Eq. (17) and the non-linear variations of soil properties in Eqs. (4)–(6). In addition, the analytical solution with constant soil properties from Wan and Mitchell [15] is compared with the analytical solution proposed in this study to analyze the effect of the non-linear variations of soil properties. The basic soil parameters used in the analytical solutions and numerical models are listed in Table 2.

4.1. Effect of the weighted factor

In order to solve u from Eq. (16), a weighted average value for $(Q + C_0 Z)$ is introduced in Eq. (20). In fact, the term $Q + C_0 Z$ can be expressed as following,

$$Q + C_0 \cdot Z = (1 - \beta W)^{1 + C_c/N - C_c/M} \quad (26)$$

As afore analyzed, when $t=0$, $u = p_0$, and $Q + C_0 Z = (1 + p_0/\sigma_0)^{(C_c/M - C_c/N - 1)}$, and when t approaches infinity, $Q + C_0 Z = 1 + C_0 Z$. Note that the value of the term $(Q + C_0 Z)$ is uniform along the vertical direction at the beginning, and linearly increases from 1 at $Z=0$ to $1 + C_0$ at $Z=1$ at the end of electro-osmosis. The average value for this term is $(1 + p_0/\sigma_0)^{(C_c/M - C_c/N - 1)}$ and $1 + 0.5C_0$ at the beginning and end of electro-osmosis respectively, and $(Q + C_0 Z)_{av}$ in Eq. (20) is actually a weighted average value along the time scale. The weighted factor ε was assumed to be 0.5 in the consolidation theory from Lekha et al. [33], however, most of

previous studies about electro-osmosis indicated that the variation of pore water pressure was highly nonlinear, therefore a weighted factor of 0.5 may not be appropriate for electro-osmotic consolidation in this study.

In order to study the change in $(Q + C_0 Z)$ during electro-osmotic consolidation and evaluate the value of ε , numerical simulations are performed considering the non-linear variations of soil properties (Eqs. (4)–(6)), and sensitivity analysis is conducted to study the effect of different factors (initial hydraulic and electro-osmosis conductivities, voltage, surcharge preloading, initial effective stress, compression index, M and N) by changing one parameter and keeping others remain as they are in Table 2 in the numerical

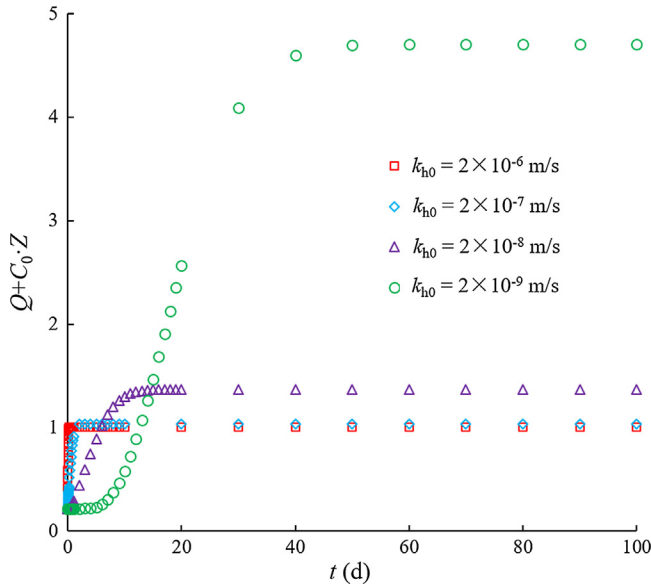


Fig. 4. Development of $(Q + C_0 \cdot Z)$ for varied hydraulic conductivity.

simulations. Fig. 4 shows the development of the values of $(Q + C_0 \cdot Z)$ with different initial hydraulic conductivities. Since the value of $(Q + C_0 \cdot Z)$ increases rapidly at the beginning and gradually becomes stable, the value of ε should be smaller than 0.5 according to the area equivalent principle. Taking the time for $U = 99.0\%$ as the end of electro-osmotic consolidation, the weighted factor ε can be estimated as 0.24, 0.29, 0.34, and 0.43 for $k_{h0} = 2 \times 10^{-6}$, 2×10^{-7} , 2×10^{-8} , and 2×10^{-9} m/s respectively. Fig. 5 further displays the estimated weighted factor under different soil parameters. The value of ε increases with the increase in k_{e0} and the decrease in k_{h0} , and the impacts of other factors are smaller than that of k_{h0} and k_{e0} . According to Fig. 5, the value of ε is 0.33 when soil parameters listed in Table 2 are adopted. For a soil with different initial hydraulic and electro-osmosis conductivities, the value of ε can be adjusted according to Fig. 5. It is worth noting that for most natural clays, k_{e0} is generally in the range of $1 \times 10^{-8} \sim 1 \times 10^{-9} \text{ m}^2/(\text{V s})$, therefore the effect of k_{e0} is small and the value of ε is mainly dominated by k_{h0} . For example, for a clay with relative large k_{h0} (1×10^{-7} m/s), the value of ε is assessed to be about 0.30, while for a clay with relative small k_{h0} (1×10^{-9} m/s), the value of ε is about 0.45. Therefore, for a natural clay submitted to electro-osmosis, the value of ε is generally in

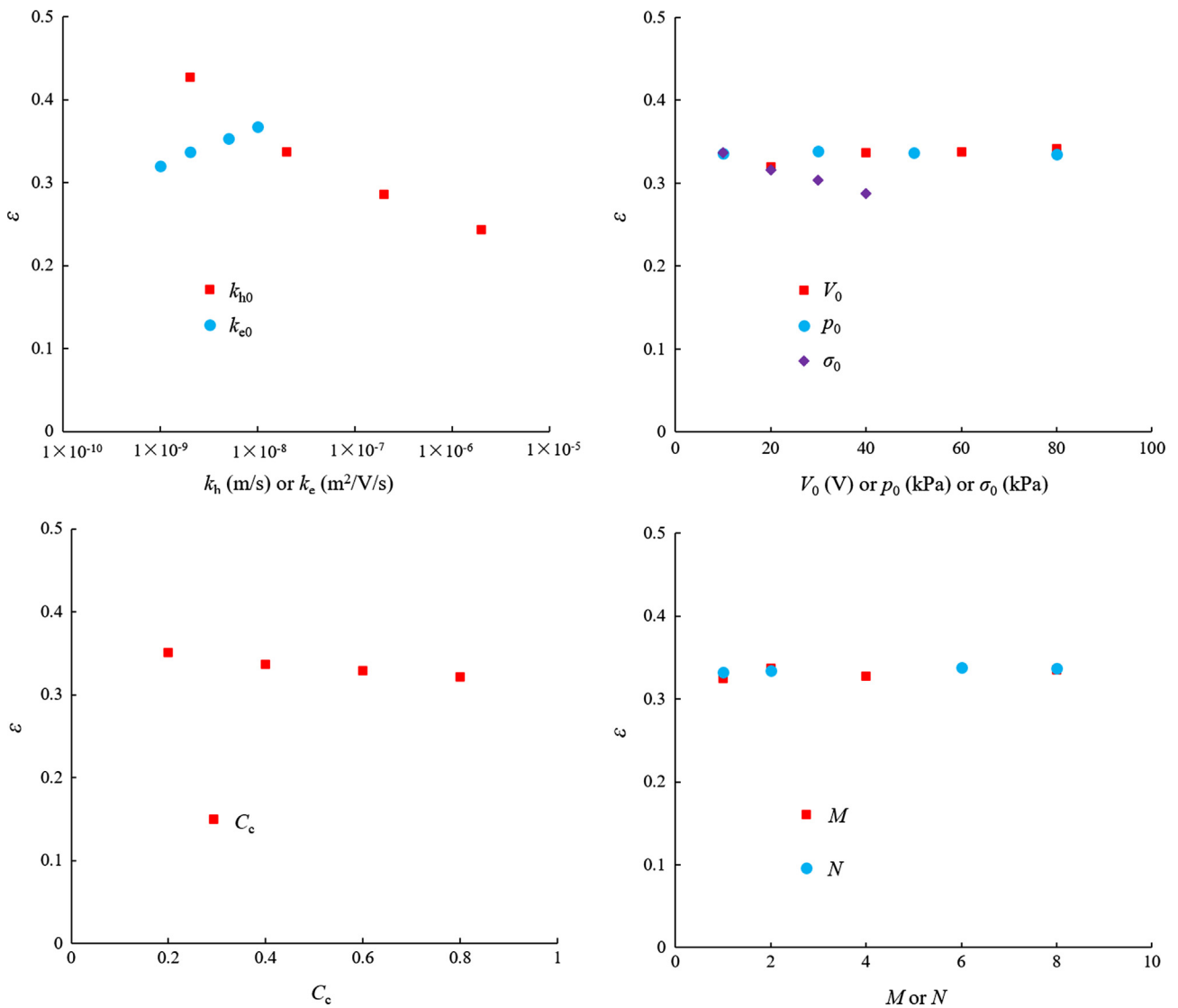


Fig. 5. Values of the weighted factor for different soil properties and electro-osmosis conditions.

the range of 0.30–0.45, and can be estimated relatively precisely according to the initial hydraulic conductivity k_{h0} , i.e., larger ε for smaller k_{h0} and vice versa.

The effect of the weighted factor ε is further shown in Fig. 6, which compares the excess pore water pressure at different depths ($z/H = 0.1$, $z/H = 0.5$ and $z/H = 1.0$) from the analytical solution and the numerical results respectively. Due to electro-osmosis, the excess pore water pressure in the model decreases from 50 kPa to a negative value at the end of the consolidation process, which is about -4.6 kPa at $z/H = 0.1$, -23.2 kPa at $z/H = 0.5$ and -46.8 kPa at $z/H = 1.0$. The development of u is highly dependent on the value of ε . With the increase in ε from 0 to 1.0, u increases during the electro-osmotic consolidation process, and $\varepsilon = 0.33$ gives the best agreement with the numerical results for the examined conditions. Therefore, the value of $\varepsilon = 0.33$ is used in the following analysis.

4.2. Excess pore water pressure

According to Eqs. (5) and (6), the higher the value of M and N , the less sensitive of k_h and k_e to the change in void ratio, and when M and N approaches infinity, k_h and k_e are actually constant during the consolidation process. Therefore, in order to examine the effect of the non-linear variation of k_h , the value of N is set to be 100, and

the value of M is set to be 100 to investigate the effect of the non-linear variation of k_e .

Fig. 7 shows the comparison of the analytical solutions and the numerical results at different depths, with $C_c = 0.2$, $N = 100$ and M varying from 0.5 to 8.0. With the increase in M , the excess pore water pressure from Wan and Mitchell [15] remains constant, while the results from the analytical solution in this study and the numerical simulation decrease. For all the M values analyzed here, the excess pore water pressure calculated from the analytical solution in this study agrees well with that from the numerical simulation, while the results from Wan and Mitchell [15] is smaller, especially at the bottom of the model. According to previous theories about electro-osmotic consolidation, the effects of k_h and k_e on the development of excess pore water pressure are different, specifically, a larger k_e results in a larger negative excess pore water pressure (under constant k_h), while a larger k_h induces a smaller one (under constant k_e). Because k_h gradually decreases during electro-osmotic consolidation, the calculated value of the ultimate excess pore water pressure in this study is larger than that from Wan and Mitchell [15]. With the increase in M , the effect of the non-linear variation of the hydraulic conductivity decreases and the difference between the calculated excess pore water pressure from this study and that from Wan and Mitchell [15] decreases.

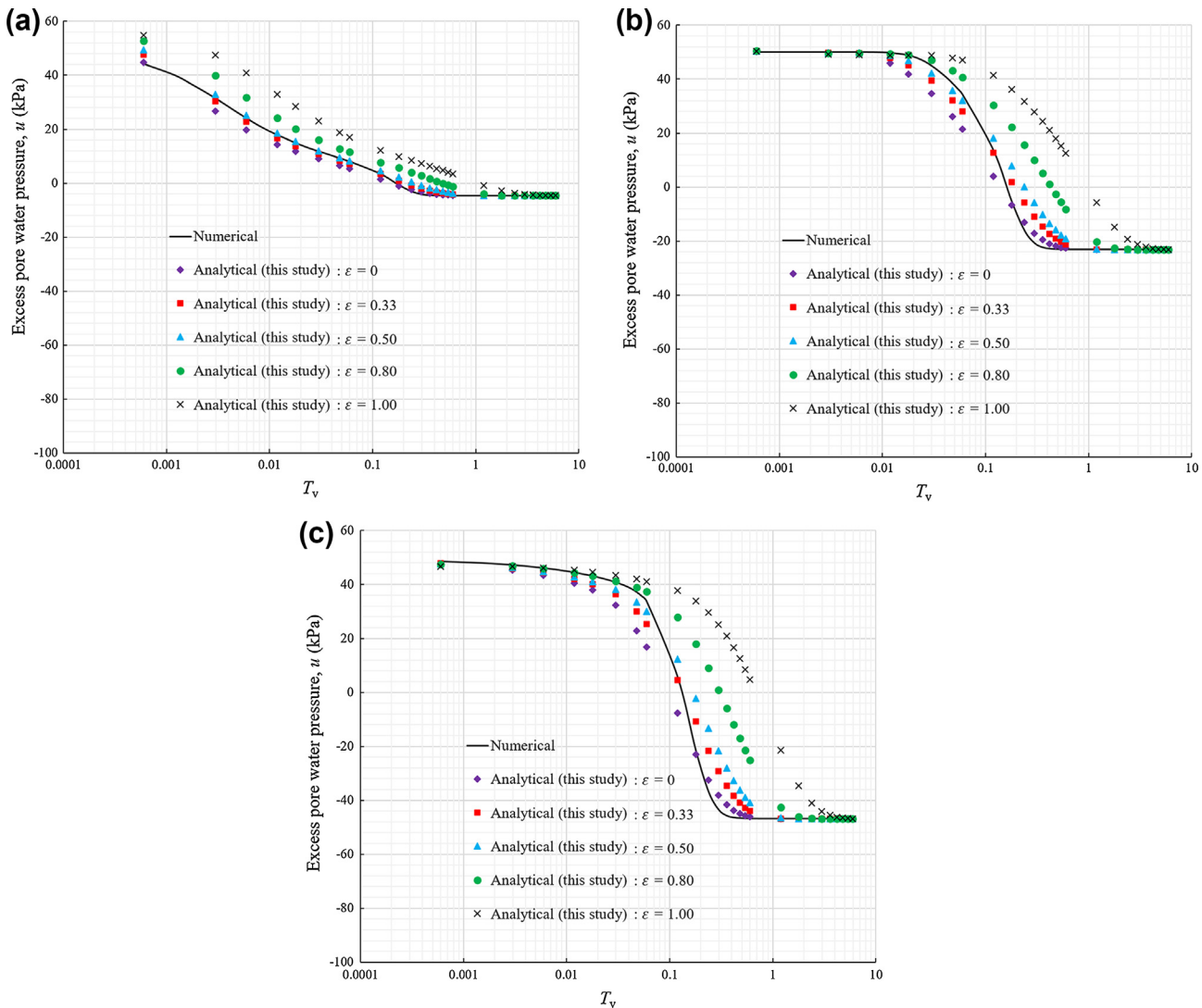


Fig. 6. Effect of the weighted factor on the development of the excess pore water pressure: (a) $z/H = 0.1$; (b) $z/H = 0.5$; and (c) $z/H = 1.0$.

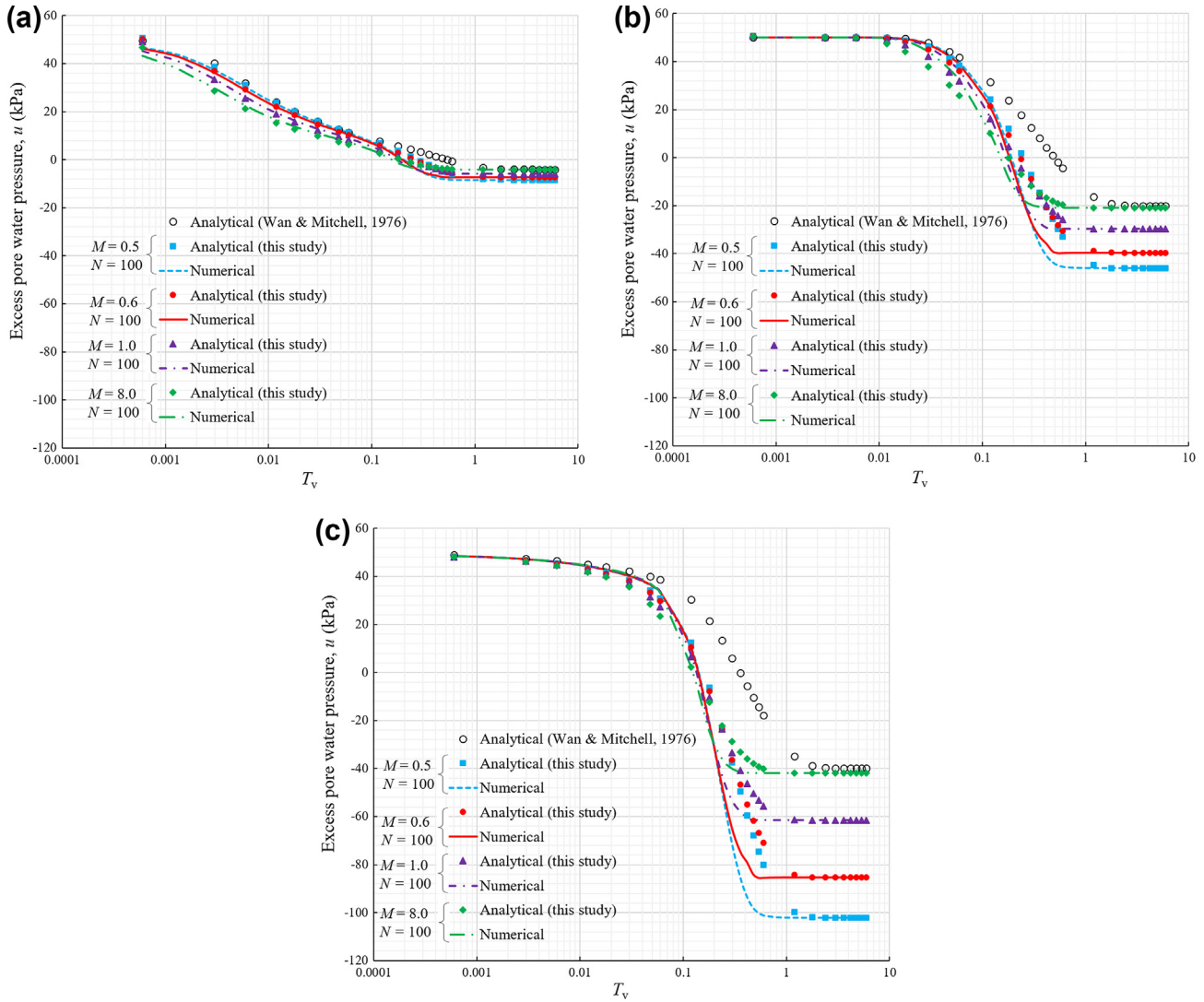


Fig. 7. Effect of the non-linear variation of hydraulic conductivity on the development of the excess pore water pressure: (a) $z/H = 0.1$; (b) $z/H = 0.5$; and (c) $z/H = 1.0$.

Fig. 8 illustrates the development of the excess pore water pressure with $C_c = 0.2$, $M = 100$ and N varying from 0.5 to 8.0. The excess pore water pressure calculated from Wan and Mitchell [15] remains constant, while the results from the presented analytical solution and the numerical simulation increase with the increase in N . For all the N values analyzed here, the results from the presented analytical solution in this study agree well with that from the numerical simulation. Since a larger k_e leads to a larger excess pore water pressure, the value of the ultimate pore water pressure in this study is smaller than that from Wan and Mitchell [15]. With the increase in N , the effect of the non-linear variation of the electro-osmosis conductivity decreases and the calculated excess pore water pressure increases.

The change in the coefficient of consolidation C_v during electro-osmotic consolidation depends on the values of C_c and M . Lekha et al. [33] indicated that if $C_c = M$, C_v remained constant, if $C_c > M$, C_v decreased with the decrease in the void ratio, and if $C_c < M$, C_v increased. For the cases examined here, $C_c < M$, and therefore C_v increases during the consolidation process. As a result, the development of the excess pore water pressure calculated from the present analytical solution is faster than that from Wan and Mitchell [15] as shown in Figs. 7 and 8.

Fig. 9 displays the effect of C_c with $M = 2.0$ and $N = 8.0$. Since the change in C_c leads to the change in C_v and T_v , t is used as the

horizontal ordinate instead of T_v . With the increase in C_c , the soil compressibility increases and C_v decreases, therefore the development of the excess pore water pressure becomes slower, both for the analytical solution in this study and that from Wan and Mitchell [15]. When the soil properties are assumed constant, the value of the ultimate negative excess pore water pressure is independent of C_c , while when the non-linear variations of soil properties are considered, the value of the ultimate negative excess pore water pressure increases with the increase in C_c . With a higher C_c , the soil is more compressible and the changes in k_h and k_e are larger during electro-osmotic consolidation, which means that the effect of the non-linear variations of soil properties is more significant. According to Figs. 7 and 8, the non-linear variation of k_h results in a larger value of the ultimate excess pore water pressure, while the non-linear variation of k_e leads to a smaller one. For the given values of k_h , k_e , M and N , the effect of the non-linear variation of k_h is more remarkable than that of k_e . As a result, a larger value of the ultimate excess pore water pressure is obtained with a higher C_c .

4.3. Degree of consolidation

The comparison of the degree of consolidation is displayed in Figs. 10–12. The effects of M , N and C_c are analyzed, respectively. Similar to that of the excess pore water pressure, with the change

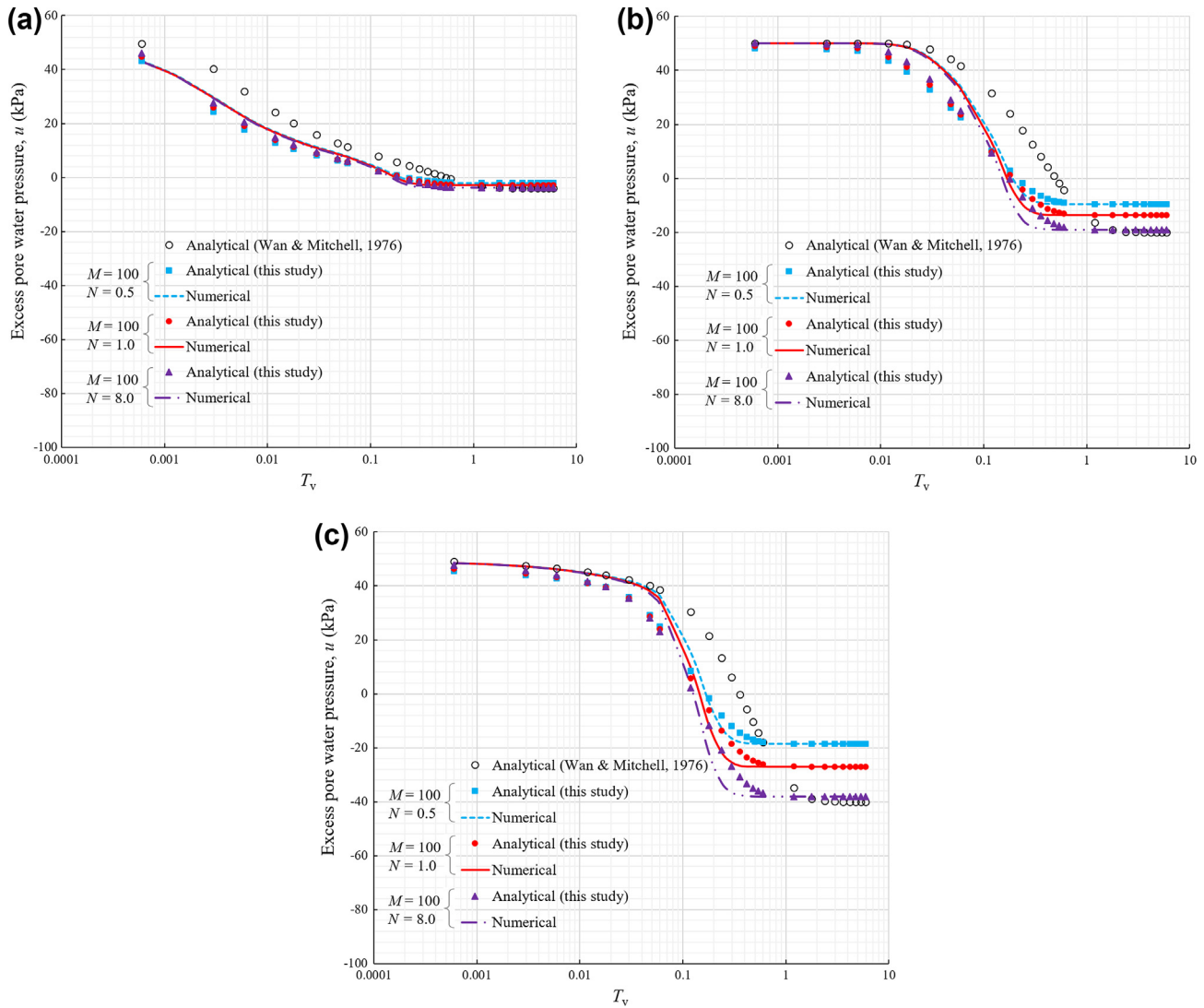


Fig. 8. Effect of the non-linear variation of electro-osmosis conductivity on the development of the excess pore water pressure: (a) $z/H = 0.1$; (b) $z/H = 0.5$; and (c) $z/H = 1.0$.

in M and N , the degree of consolidation obtained from Wan and Mitchell [15] remains constant, while the results from the present analytical solution and the numerical simulation increase with the increase in M , and decrease with the increase in N . As mentioned before, the decrease in k_h during the consolidation process is smaller with a larger M , therefore the value of the ultimate excess pore water pressure is smaller and the coefficient of consolidation is larger. As a result, the degree of consolidation is larger for a larger M . Opposite to k_h , the value of the ultimate excess pore water pressure is positively related to k_e , and therefore is larger for a larger N , hence the degree of consolidation decreases with the increase in N .

According to the above analysis, the non-linear variation of k_h influences not only the excess pore water pressure but also the coefficient of consolidation, while the non-linear variation of k_e only affects the excess pore water pressure. Consequently, the impact of the non-linear variation of k_h is larger than that of k_e , both for the excess pore water pressure and the degree of consolidation as shown in Figs. 7, 8, 10 and 11.

The increase in C_v during the consolidation process is neglected since the soil compressibility is assumed constant in the theory of Wan and Mitchell [15], therefore the degree of consolidation calcu-

lated from Wan and Mitchell [15] is smaller than that from the present analytical solution and the numerical simulation (Figs. 10 and 11). The impact of C_c is further shown in Fig. 12. Compared to the results of Wan and Mitchell [15], the degree of consolidation obtained from the present analytical simulation is larger since C_v increases during the consolidation process when $C_c < M$. With the increase in C_c , C_v decreases and therefore the degree of consolidation decreases.

Figs. 7–9 illustrates that the distribution of the excess pore water pressure during electro-osmotic consolidation calculated from the present analytical solution agrees well with that from the numerical simulation, and Figs. 10–12 further indicates that the degree of consolidation can also be well predicted with the present analytical solution.

In the above analysis about the excess pore water pressure and the degree of consolidation, the values of M and N are set to change from 0.5 to 100 in order to analyze the impact of the non-linear variations of k_h and k_e respectively. In fact, according to previous studies, k_e is generally less sensitive to the change in void ratio compared to k_h [5, 7, 39, 40]. Therefore, the above-mentioned case of $M < N$ is more common in practical situation.

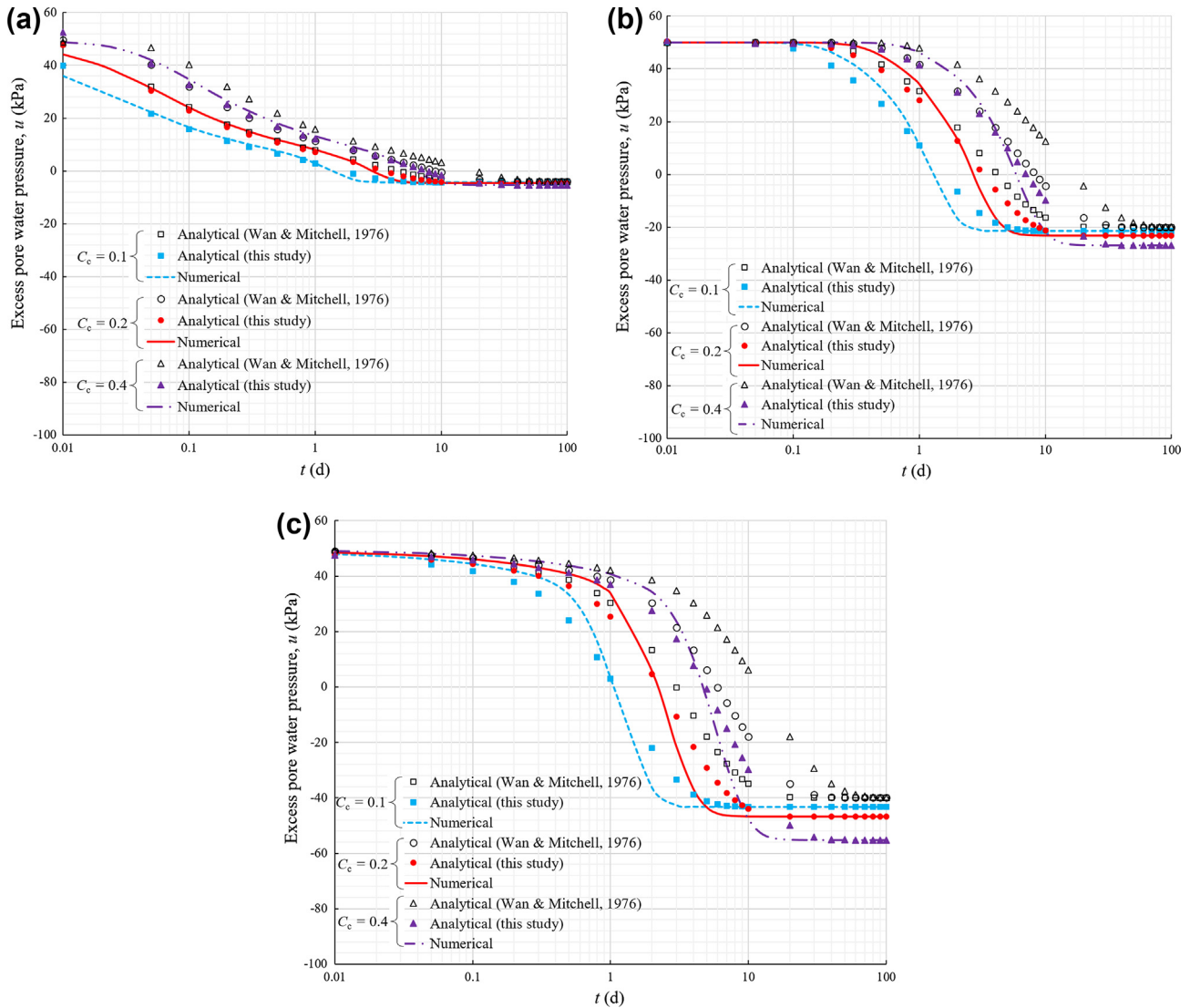


Fig. 9. Effect of the non-linear variation of soil compressibility on the development of the excess pore water pressure: (a) $z/H = 0.1$; (b) $z/H = 0.5$; and (c) $z/H = 1.0$.

4.4. Ultimate excess pore water pressure

Due to the introduction of ε and the simplification of the governing equation from Eqs. (16)–(21), there is a slight deviation between the analytical solution and the numerical simulation as illustrated by Fig. 6. However, it is worth noting that the ultimate excess pore water pressure achieved after $T_v = 1$ is almost the same in the analytical solution and the numerical simulation, regardless of the value of ε . Figs. 7–9 further demonstrate that although the excess pore water pressure (absolute value) obtained from the proposed analytical solution is slightly different from the numerical simulation, the ultimate excess pore water pressure calculated from the two methods agrees quite well with each other. It seems that the ultimate excess pore water pressure calculated from Eq. (23) with t approaching infinity is not affected by the introduction of the weighted average value for $(Q + C_0 Z)$. In fact, the solution for the ultimate excess pore water pressure can be directly derived from the equilibrium of the hydraulic flow and electro-osmosis flow at the end of electro-osmotic consolidation process.

At equilibrium, the pore water flow in the soil mass is 0, and the following equation can be obtained,

$$\frac{k_h}{\gamma_w} \cdot \frac{\partial u_{ult}}{\partial Z} + k_e \cdot \frac{\partial V}{\partial Z} = 0 \quad (27)$$

in which u_{ult} denotes the excess pore water pressure at the end of electro-osmotic consolidation.

Making the substitutions $W_{ult} = u_{ult}/p_0$ and $Z = z/H$,

$$\frac{\partial W_{ult}}{\partial Z} + \frac{k_e \cdot \gamma_w}{k_h \cdot p_0} \cdot \frac{\partial V}{\partial Z} = 0 \quad (28)$$

Substituting Eqs. (12) and (13) into Eq. (28), the following equation is obtained,

$$\frac{\partial W_{ult}}{\partial Z} = -\frac{k_{e0} \cdot \gamma_w}{k_{h0} \cdot p_0} \cdot \frac{\partial V}{\partial Z} \cdot \left[\left(1 + \frac{p_0}{\sigma_p} \right) \cdot (1 - \beta W_{ult}) \right]^{C_c/M - C_c/N} \quad (29)$$

Defining Q_{ult} as,

$$Q_{ult} = (1 - \beta W_{ult})^{1 + C_c/N - C_c/M} \quad (30)$$

Then Eq. (29) can be written as,

$$\frac{\partial Q_{ult}}{\partial Z} = \beta \cdot \left(1 + \frac{C_c}{N} - \frac{C_c}{M} \right) \cdot \left(1 + \frac{p_0}{\sigma_p} \right)^{C_c/M - C_c/N} \cdot \frac{k_{e0} \cdot \gamma_w}{k_{h0} \cdot p_0} \cdot \frac{\partial V}{\partial Z} = C_0 \quad (31)$$

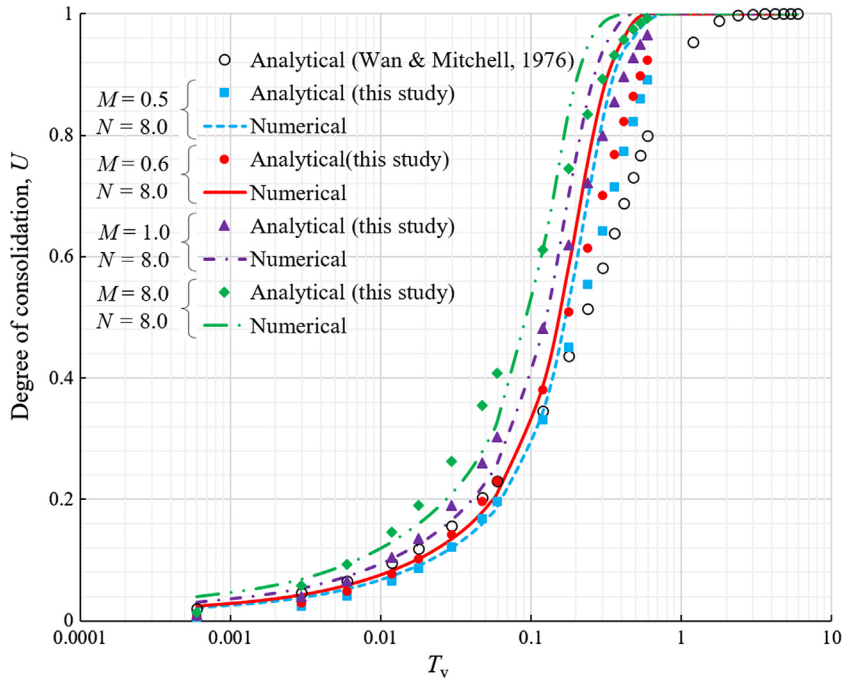


Fig. 10. Effect of the non-linear variation of hydraulic conductivity on the degree of consolidation.

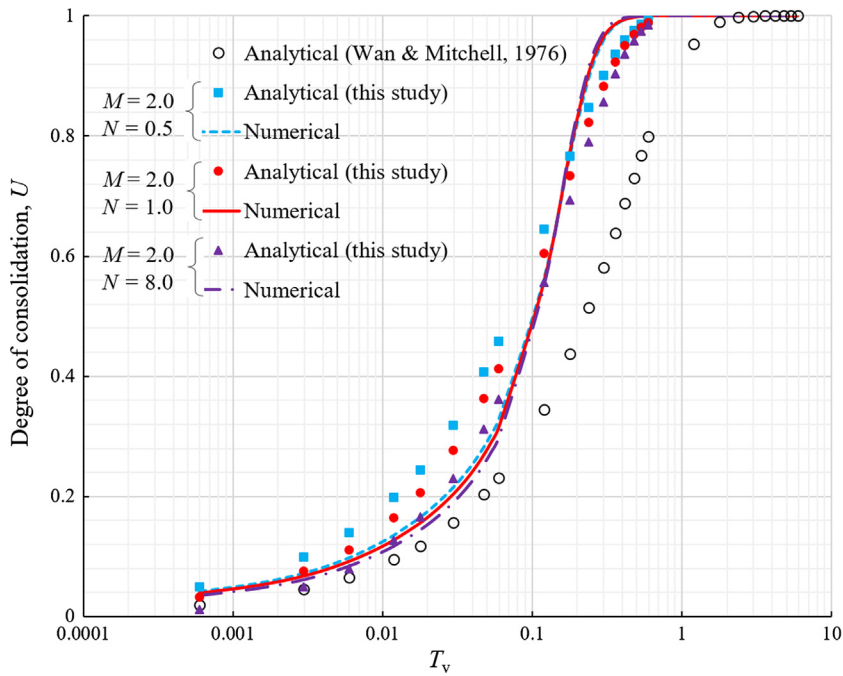


Fig. 11. Effect of the non-linear variation of electro-osmosis conductivity on the degree of consolidation.

Integrating Eq. (31) in Z direction and considering the boundary condition that both the excess pore water pressure and voltage are 0 at the cathode, the solution for u_{ult} can be obtained,

$$u_{ult} = \frac{p_0}{\beta} \cdot [1 - (1 + C_0 \cdot Z)^{1/(1+C_c/N-C_c/M)}] \quad (32)$$

The above derivation indicates that the ultimate excess pore water pressure can be solved from Eq. (27) without any simplification or assumption. In fact, the solution for u_{ult} shown in Eq. (32) can also be obtained from the solution for u (Eq. (23)) by making t infinity.

According to Eq. (32), the ultimate excess pore water pressure (absolute value) decreases with the increase in M and increases with the increase in N (Figs. 7 and 8). The non-linear variation of k_h leads to a larger u_{ult} , while the non-linear variation of k_e results in a smaller one. When both M and N approach infinity or $M = N$, the effect of the non-linear variations of k_h and k_e is eliminated or balanced by each other, and therefore the analytical solution for the ultimate excess pore water pressure degrades to the equation proposed by Esrig [14] and Wan and Mitchell [15],

$$u_{ult} = -\frac{k_{e0} \cdot \gamma'_w \cdot V}{k_{h0}} \quad (33)$$

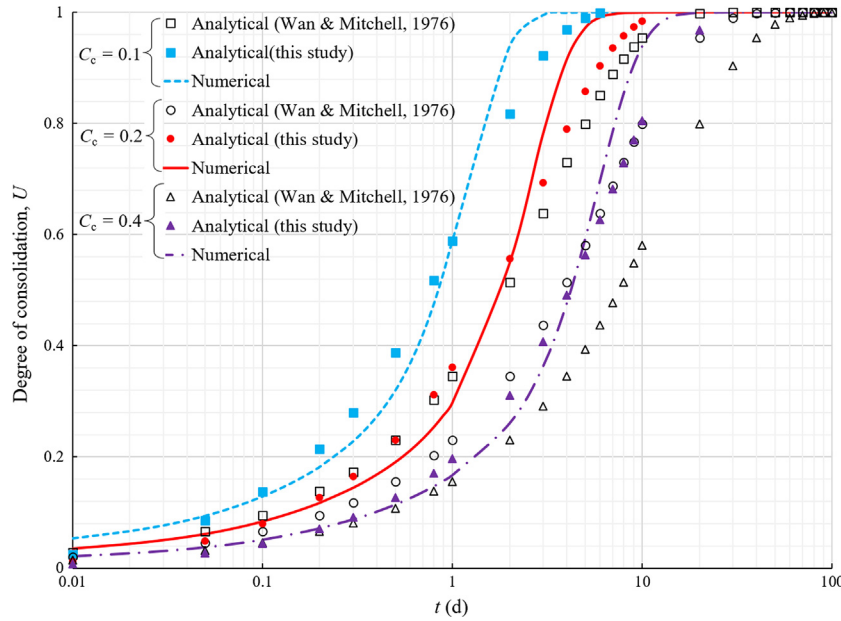


Fig. 12. Effect of the non-linear variation of soil compressibility on the degree of consolidation.

5. Summary and conclusions

In this study, the non-linear variations of soil compressibility, hydraulic conductivity and electro-osmosis conductivity during the consolidation process are analyzed through laboratory tests and incorporated in a 1D model for electro-osmotic consolidation. The analytical solutions for the excess pore water pressure and degree of consolidation are derived, and compared with the results from numerical simulations for verification. Both the analytical solutions and numerical results are further compared with previous solutions from Wan and Mitchell [15] to analyze the effect of the non-linear variations of soil properties.

With the decrease in void ratio, both the hydraulic conductivity and electro-osmosis conductivity decrease, and linear relationships between the logarithm of them and the void ratio are found from the experiment results. Two empirical formulas are further developed to account for the non-linear variations of the hydraulic conductivity and electro-osmosis conductivity.

The non-linear variations of soil properties show remarkable impact on the development of the excess pore water pressure and degree of consolidation during electro-osmotic consolidation. Specifically, the non-linear variation of hydraulic conductivity results in a larger excess pore water pressure, while the non-linear variation of electro-osmosis conductivity leads to a smaller one. The more sensitive of hydraulic and electro-osmosis conductivities to the change in void ratio, the more significant the impact they show. With the increase in the initial soil compressibility, the development of the excess pore water pressure becomes slower, and the impact of the nonlinear variation of the hydraulic and electro-osmosis conductivities becomes more remarkable since the changes in the void ratio are larger with higher compressibility.

The coefficient of consolidation is related to soil compressibility and hydraulic conductivity. When the decrease in soil compressibility is larger than the decrease in hydraulic conductivity, the coefficient of consolidation increases during the consolidation process, therefore the degree of consolidation calculated from the present analytical solution is larger than that from previous solutions with constant soil properties.

The present analytical results agree well with the numerical results, both for the excess pore water pressure and the degree of consolidation. Compared to the previous analytical solutions, the

newly proposed analytical solutions give more accurate predictions, and can be used to analyze the consolidation behavior of soil treated by electro-osmotic consolidation.

Acknowledgments

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Appendix A. Details of the derivation of the 1D model for electro-osmotic consolidation with variable compressibility, hydraulic conductivity and electro-osmosis conductivity

The governing equation and the boundary conditions and the initial condition can be expressed as,

$$\begin{cases} \frac{\partial Q}{\partial t_e} = \frac{\partial^2 Q}{\partial Z^2} + \alpha \cdot \frac{\partial Q}{\partial Z} \cdot \frac{\partial Q}{\partial Z} + \theta \cdot \frac{\partial Q}{\partial Z} \\ Z = 0, u = 0 \Rightarrow Q = 1 \\ Z = 1, \frac{k_h}{\gamma_w} \frac{\partial u}{\partial z} + k_e \frac{\partial V}{\partial z} = 0 \Rightarrow \frac{\partial Q}{\partial Z} = 0 \\ t = 0, u = p_0 \Rightarrow Q = (1 - \beta)^{1+C_c/N-C_c/M} - C_0 \cdot Z \end{cases} \quad (A1)$$

Defining a new variable R as,

$$R = e^{\alpha \cdot (Q-1)} - 1 \quad (A2)$$

Substituting Eq. (A2) into Eq. (A1) and the following equation can be obtained,

$$\begin{cases} \frac{\partial R}{\partial t_e} = \frac{\partial^2 R}{\partial Z^2} + \theta \cdot \frac{\partial R}{\partial Z} \\ Z = 0, R = 0 \\ Z = 1, \frac{\partial R}{\partial Z} = 0 \\ t = 0, R = e^{\alpha \left[\left(1 + \frac{p_0}{\sigma_0}\right)^{C_c/M-C_c/N-1} - C_0 Z - 1 \right]} - 1 = e^{\alpha \left[\left(1 + \frac{p_0}{\sigma_0}\right)^{C_c/M-C_c/N-1} - 1 \right]} \cdot e^{-\alpha C_0 Z} - 1 \end{cases} \quad (A3)$$

Using the separating method and R can be expressed as,

$$R(Z, T_e) = T(T_e) \cdot Z(Z) \quad (A4)$$

Substituting Eq. (A4) into the governing equation in Eq. (A3),

$$\begin{aligned} T'(T_e)Z(Z) &= T(T_e)Z''(Z) + \theta T(T_e)Z'(Z) \Rightarrow \frac{T'(T_e)}{T(T_e)} = \frac{Z''(Z) + \theta \cdot Z'(Z)}{Z(Z)} \\ &= -\lambda \Rightarrow \begin{cases} T'(T_e) + \lambda T(T_e) = 0 \\ Z''(Z) + \theta \cdot Z'(Z) + \lambda Z(Z) = 0 \end{cases} \end{aligned} \quad (\text{A5})$$

$T(T_e)$ can be solved as,

$$T(T_e) = D_n e^{-\lambda T_e} \quad (\text{A6})$$

$Z(Z)$ can be solved as,

$$\begin{aligned} Z(Z) &= e^{-\frac{1}{2}\theta Z} [F_1 \sin\left(\frac{1}{2}\xi Z\right) + F_2 \cos\left(\frac{1}{2}\xi Z\right)] \\ \xi^2 &= 4\lambda - \theta^2 \end{aligned} \quad (\text{A7})$$

Considering the boundary conditions, $F_2 = 0$ and ξ should satisfy the following,

$$\tan\left(\frac{\xi}{2}\right) = \frac{\xi}{\theta} \quad (\text{A8})$$

The solutions for ξ can be solved from Eq. (A8) and mark them as ξ_n ($n = 1, 2, 3, 4, 5, \dots$).

Therefore, $Z(Z)$ can be further derived as,

$$Z(Z) = \sum_{n=1}^{\infty} e^{-\frac{1}{2}\theta Z} \cdot F_{1n} \sin\left(\frac{1}{2}\xi_n Z\right) \quad (\text{A9})$$

Then,

$$\begin{aligned} R &= \sum_{n=1}^{\infty} G_n e^{-\lambda_n T_e} \cdot e^{-\frac{1}{2}\theta Z} \cdot \sin\left(\frac{1}{2}\xi_n Z\right) \\ \frac{\xi_n^2 + \theta^2}{4} &= \lambda_n \end{aligned} \quad (\text{A10})$$

According to the initial condition, G_n can be calculated as,

$$\begin{aligned} G_n &= \frac{I \cdot \frac{2}{\xi_n} - I \cdot e^J \cdot \frac{2}{\xi_n} \cdot \frac{\theta}{\sqrt{\theta^2 + \xi_n^2}} + I \cdot J \cdot e^J \cdot \frac{4}{\xi_n} \cdot \frac{\xi_n}{\sqrt{\theta^2 + \xi_n^2}}}{\left(1 + J^2 \cdot \frac{4}{\xi_n^2}\right) \cdot \left(\frac{1}{2} - \frac{\theta}{\theta^2 + \xi_n^2}\right)} \\ &\quad - \frac{\frac{2}{\xi_n} - e^{\frac{1}{2}\theta} \cdot \frac{2}{\xi_n} \cdot \frac{\theta}{\sqrt{\theta^2 + \xi_n^2}} + \frac{1}{2} \theta \cdot e^{\frac{1}{2}\theta} \cdot \frac{4}{\xi_n} \cdot \frac{\xi_n}{\sqrt{\theta^2 + \xi_n^2}}}{\left(1 + \frac{\theta^2}{\xi_n^2}\right) \cdot \left(\frac{1}{2} - \frac{\theta}{\theta^2 + \xi_n^2}\right)} \end{aligned} \quad (\text{A11})$$

where

$$I = e^{\alpha \left[\left(1 + \frac{p_0}{\sigma_0}\right)^{C_c/M - C_c/N - 1} \right]} \quad J = \frac{1}{2} \theta - \alpha C_0 \quad (\text{A12})$$

And Q can be expressed as,

$$Q = \frac{1}{\alpha} \ln \left[\sum_{n=1}^{\infty} G_n e^{-\lambda_n T_e} \cdot e^{-\frac{1}{2}\theta Z} \cdot \sin\left(\frac{1}{2}\xi_n Z\right) + 1 \right] + 1 \quad (\text{A13})$$

Then, the excess pore water pressure u can be derived as,

$$u = \frac{p_0}{\beta} - \frac{p_0}{\beta} \cdot \left\{ \frac{1}{\alpha} \ln \left[\sum_{n=1}^{\infty} G_n e^{-\lambda_n T_e} \cdot e^{-\frac{1}{2}\theta Z} \cdot \sin\left(\frac{1}{2}\xi_n Z\right) + 1 \right] + 1 + C_0 Z \right\}^{1/(1+C_c/N - C_c/M)} \quad (\text{A14})$$

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