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Technical Notes

Explicit Role of Viscosity in Generating Lift

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Nomenclature

C_d	=	drag coefficient
C_l	=	lift coefficient
C_p	=	pressure coefficient
c	=	chord length, m
D	=	drag, N
$\boldsymbol{F}$	=	fluid-mechanic force or aerodynamic force, N
f_Ω	=	boundary enstrophy flux, $\mathrm{kg \cdot m^{-2} \cdot s^{-3}}$
L	=	lift, N
L_{p_e}	=	pressure lift, N
L_{vor}	=	vortex lift, N
$\boldsymbol{l}$	=	Lamb vector; $\boldsymbol{u} \times \boldsymbol{\omega}$, $\mathrm{m \cdot s^{-2}}$
N	=	normal force to the flat plate, N
p	=	pressure, Pa
q_∞	=	freestream dynamic pressure; $\rho U_\infty^2/2$, Pa
Re_c	=	Reynolds number; $U_\infty c/\nu$
$U(x)$	=	outer flow velocity, $\mathrm{m \cdot s^{-1}}$
U_{ref}	=	reference velocity, $\mathrm{m \cdot s^{-1}}$
U_∞	=	freestream velocity, $\mathrm{m \cdot s^{-1}}$
$\boldsymbol{u}$	=	fluid velocity, $\mathrm{m \cdot s^{-1}}$
$u(x, y)$	=	velocity in boundary layer, $\mathrm{m \cdot s^{-1}}$
x, y	=	coordinates on the flat plate, m
$\bar{x}$	=	normalized coordinate along the flat plate; x/c
α	=	angle of attack, rad
Γ_g	=	generalized circulation, $\mathrm{m^2 \cdot s^{-1}}$
ΔC_p	=	pressure coefficient difference across the flat plate
Δp	=	pressure loading the flat plate, Pa
μ	=	dynamic viscosity of fluid, $\mathrm{kg \cdot m^{-1} \cdot s^{-1}}$
ν	=	kinematic viscosity of fluid, $\mathrm{m^2 \cdot s^{-1}}$
ρ	=	fluid density, $\mathrm{kg \cdot m^{-3}}$
$\boldsymbol{\tau}$	=	skin friction, Pa
Ω	=	enstrophy; $\lvert\omega\rvert^2/2$, $\mathrm{s^{-2}}$
$\boldsymbol{\omega}$	=	vorticity; $\nabla \times \boldsymbol{u}$, $\mathrm{s^{-1}}$

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I. Introduction

THIS Note tries to elucidate the subtle but critical role of the fluid viscosity in generating the aerodynamic lift. The viscous origin of the lift has not been widely understood or sufficiently highlighted in the literature. In classical aerodynamics textbooks [1–4], the lift theory of an airfoil is usually developed based on the classical potential-flow theory, in which the Kutta–Joukowski (K-J) theorem is used as a key element in calculating the lift. However, the airfoil circulation in the K-J theorem cannot be automatically determined in the potential-flow framework alone because the vorticity cannot be physically generated in an inviscid flow. This fundamental difficulty is intrinsically related to D'Alembert's paradox stating that the integrated pressure force of a body is zero in a steady inviscid irrotational incompressible flow [5,6]. To circumvent this problem, the Kutta condition is imposed at the airfoil trailing edge to determine the circulation. To a great extent, the success of the classical circulation theory in predicting the lift depends on the clever application of the Kutta condition and its generalized forms as the phenomenological models of the viscous-flow effect on the lift generation [7]. The important implication of D'Alembert's paradox is that the generation of the lift and drag of a moving body must be a result of a viscous flow no matter how small the fluid viscosity is. The viscous origin of the lift has been recognized, which has to be essentially found in the viscous-flow framework [8–10]. However, an intriguing question is whether there is a direct and explicit connection between the fluid viscosity and the integrated pressure force contributing the lift. This problem is the central topic investigated from a new perspective here.

II. Force Expression

For an incompressible viscous flow over a stationary surface, an exact relation between the surface pressure gradient ∇p and the skin-friction vector $\boldsymbol{\tau}$ can be derived from the Navier–Stokes (NS) equations [11], and it is written as

$$\boldsymbol{\tau} \cdot \nabla p = \mu f_\Omega \tag{1}$$

where the scalar quantity is defined as

$$f_\Omega = \mu \partial\Omega/\partial n - 2\mu\kappa_\omega\Omega \tag{2}$$

μ is the dynamic viscosity of the fluid, $\Omega = |\boldsymbol{\omega}|^2/2$ is the enstrophy, $\boldsymbol{\omega}$ is the vorticity, κ_ω is the curvature of the boundary vorticity line, and $\partial/\partial n$ is the derivative along the unit normal outward vector $\boldsymbol{n}$ of the surface. Equation (1) holds instantaneously for a stationary surface. In Eq. (2), the first term $\mu\partial\Omega/\partial n$ is the boundary enstrophy flux (BEF), and the second term is the curvature-induced contribution. In general, f_Ω is dominated by the BEF, whereas the curvature term can be neglected. The BEF is an intriguing quantity that is particularly related to the topological features such as isolated critical points and separation attachment lines in a skin-friction field [11].

Along a skin-friction line (called a $\boldsymbol{\tau}$ line), Eq. (1) is rewritten as $\mathrm{d}p/\mathrm{d}s = \boldsymbol{s} \cdot \nabla p = \mu f_\Omega |\boldsymbol{\tau}|^{-1}$, where $\boldsymbol{s} = \boldsymbol{\tau}/|\boldsymbol{\tau}|$ is the unit vector along the $\boldsymbol{\tau}$ line. Therefore, the surface pressure is given by the path integral along a $\boldsymbol{\tau}$ line, i.e.,

$$p(\boldsymbol{x}) = \mu \int_{P_0(x_0)}^{P(x)} f_\Omega |\boldsymbol{\tau}|^{-1}\,\mathrm{d}s = \mu \int_{P_0(x_0)}^{P(x)} \frac{\partial|\omega|}{\partial n}\,\mathrm{d}s \tag{3}$$

where $P(\boldsymbol{x})$ and $P_0(\boldsymbol{x}_0)$ denote a point and a starting point on a $\boldsymbol{\tau}$ line, respectively. In principle, a set of the starting points $P_0(\boldsymbol{x}_0)$ from which the $\boldsymbol{\tau}$ lines are originated could be selected such that the points

$P(\boldsymbol{x})$ on a set of the $\boldsymbol{\tau}$ lines can cover densely the whole surface. The derivation of the second equality of Eq. (3) uses the skin-friction relation $\boldsymbol{\tau} = \mu\boldsymbol{\omega} \times \boldsymbol{n}$, where $\boldsymbol{n}$ is the unit normal outward vector of the surface of a body and. According to Eq. (3), the surface pressure is related to the accumulating effect of the viscous diffusion flux of the boundary vorticity magnitude $(\mu\partial|\omega|/\partial n)$ along a skin-friction line.

Equation (3) symbolically expresses a surface pressure field. Thus, the fluid-mechanic force is formally expressed as the following surface integral:

$$\begin{aligned}\boldsymbol{F} &= \oint_S (-p\boldsymbol{n} + \boldsymbol{\tau})\,\mathrm{d}S \\ &= -\mu\oint_S \boldsymbol{n}\,\mathrm{d}S \int_{P_0(x_0)}^{P(x)} \frac{\partial|\omega|}{\partial n}\,\mathrm{d}s + \mu\oint_S \boldsymbol{\omega}\times\boldsymbol{n}\,\mathrm{d}S \end{aligned} \tag{4}$$

Equation (4) explicitly describes the critical role of the fluid viscosity in generating the fluid-mechanic force. For a steady inviscid incompressible flow with $\mu = 0$, as long as $f_\Omega|\boldsymbol{\tau}|^{-1} = \partial|\omega|/\partial n$ does not become infinite as $\mu \to 0$, we have $\boldsymbol{F} = 0$ according to Eq. (4), and thus D'Alembert's paradox is naturally recovered. The first term in Eq. (4) is the integrated pressure force that is explicitly related to the viscosity. The main consequence of Eq. (4) is that the lift and drag (including the pressure and skin-friction drags) must coexist as a result of the viscous flow over an airfoil. Physically, the lift cannot be generated without the cost of generating the viscous drag at the same time.

III. Force of Flat Plate

To elucidate the connection between boundary layer and lift generation through Eq. (4), the steady low-Reynolds-number flow over a two-dimensional flat plate at a small angle of attack (AOA, denoted by α) is considered as an example [12–14]. As shown in Fig. 1, the thick boundary layers with opposite vorticity are developed on the upper and lower surfaces of the plate, and the thick wake sheds from the trailing edge. This viscous flow pattern is considerably different from the potential flow pattern over a flat plate imposed with the Kutta condition described in aerodynamics textbooks. It would be interesting to see how the viscous-flow model based on Eq. (4) differs from the classical aerodynamic models. To model the boundary layers of the flat plate, the Falkner–Skan flow with the external velocity $U(x) = ax^m$ for a viscous wedge flow is used [15], where x is the coordinate on the surface from the leading edge. The power-law exponent and the proportional coefficient are given by the piecewise functions $m = m_k = (-1)^k\alpha/(\pi - \alpha)$ and $a = a_k$, respectively, where α denotes the AOA, and the subscripts $k = 1$ and $k = 2$ denote the upper and lower surfaces, respectively. The boundary-layer velocity profile is given by $u(x, y)/U(x) = f'(\eta)$, where $f(\eta)$ is the self-similar stream function, and the prime denotes the differentiation with respect to the similarity variable $\eta = y\sqrt{(m+1)U/(2\nu x)}$. For the Falkner–Skan flow, skin friction is given by

$$\tau = \sqrt{\frac{m+1}{2}}\rho\nu^{1/2}a^{3/2}f''(0)x^{(3m-1)/2} \tag{5}$$

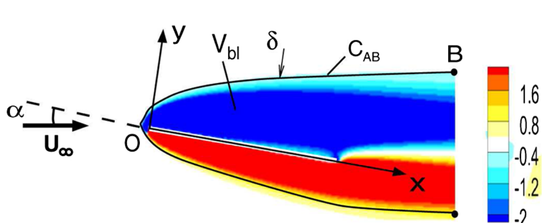

Fig. 1 Viscous flow over a flat plate, where the color bar indicates the normalized vorticity.

where ν is the kinematic viscosity. The BEF is given by

$$f_\Omega = \mu\frac{\partial\Omega}{\partial y} = -\mu\beta\left(\frac{m+1}{2}\frac{a}{\nu}\right)^{3/2} a^2 f''(0)x^{(7m-3)/2} \tag{6}$$

where the value of the second derivative $f''(0)$ at the wall is a function of m, $f'''(0) = -\beta$ from the Falkner–Skan equation is used, and $\beta = 2m/(m+1)$ is related to the wedge angle of $\pi\beta$.

When $P_0(\boldsymbol{x}_0)$ is the stagnation point at the leading edge, substitution of Eqs. (5) and (6) into Eq. (3) yields the surface pressure distributions on the upper and lower surface, i.e., $p_k(x) = p_0 + \rho a_k^2 x^{2m_k}/2$ $(k = 1, 2)$, where p_0 is the pressure at the leading edge $(x = 0)$. In this two-dimensional (2-D) case, integration along a $\boldsymbol{\tau}$ line is simply done along the x coordinate because the $\boldsymbol{\tau}$ line is aligned with the surface coordinate. Interestingly, in this case, the relevant boundary-layer parameters are canceled out in Eq. (3). Therefore, the nondimensional form of the pressure loading $\Delta p = p_2 - p_1$ is

$$\Delta C_p = \frac{\Delta p}{q_\infty} = R_q[\bar{x}^{2m_1} - (1 - \Delta C_{p,\mathrm{TE}}R_q^{-1})\bar{x}^{-2m_1}] \tag{7}$$

where $\bar{x} = x/c$ is the chordwise coordinate normalized by the chord length c; $R_q = q_{\mathrm{TE}}/q_\infty = (U_{\mathrm{ref}}/U_\infty)^2 Re_c^{2m_1}$ is the ratio between the trailing-edge and freestream dynamic pressures that are given by $q_{\mathrm{TE}} = (\rho/2)(a_1c^{m_1})^2$ and $q_\infty = (\rho/2)U_\infty^2$, respectively; U_∞ is the freestream velocity; $U_{\mathrm{ref}} = a_1(\nu/U_\infty)^{m_1}$ is a reference velocity; $Re_c = U_\infty c/\nu$ is the Reynolds number; $\Delta C_{p,\mathrm{TE}}$ is the value of ΔC_p at the trailing edge; and $m_1 = -\alpha/(\pi - \alpha)$. The ΔC_p distribution is weakly dependent of Re_c because $|m_1| \approx \alpha/\pi \ll 1$. At the leading edge, as $\bar{x} \to 0$, there is a singularity of $\Delta C_p \to \bar{x}^{2m_1}$ with $m_1 = -\alpha/(\pi - \alpha)$, and the singularity is weakened as $\alpha \to 0$, which seems physically reasonable in a viscous flow. In contrast, in the classical thin-airfoil theory, the stronger singularity $\Delta C_p \to \bar{x}^{-1/2}$ at the leading edge remains unchanged even as $\alpha \to 0$.

The normal force of the flat plate (N) is calculated by integrating Δp from $x = 0$ and $x = c$. Then, the lift and drag coefficients are approximately expressed as

$$C_l = \frac{L}{q_\infty c} \approx 2\pi\alpha F(\alpha)\cos(\alpha) \tag{8}$$

$$C_d = \frac{D}{q_\infty c} \approx 2\pi\alpha F(\alpha)\sin(\alpha) + C_{d,0} \tag{9}$$

where the nonlinear factor is defined as

$$F(\alpha) = \frac{R_q}{2}\left(1 - \frac{\alpha}{\pi}\right)\frac{4 + \Delta C_{p,\mathrm{TE}}R_q^{-1}(\pi - 3\alpha)}{(\pi - \alpha)^2 - 4\alpha^2} \tag{10}$$

and $C_{d,0}$ is the parasite drag (zero-lift drag) coefficient (the value of C_d at zero AOA in this case). The lift formula Eq. (8) is different from the linear relation $C_l = 2\pi\alpha$ given by the classical thin-airfoil theory due to the nonlinear factor $F(\alpha)$ depending on the parameters R_q and $\Delta C_{p,\mathrm{TE}}$. The parameter R_q is determined by the velocity ratio $U_{\mathrm{ref}}/U_\infty$ and the Reynolds number Re_c. In the limiting case, as $\alpha \to 0$, the asymptotic behavior of $F(\alpha) \to 2(U_{\mathrm{ref}}/U_\infty)^2/\pi^2 = 1$ is inferred because Eq. (8) should be consistent with the classical thin-airfoil theory with $\Delta C_{p,\mathrm{TE}} = 0$ in this limiting case. Therefore, the parameter $U_{\mathrm{ref}}/U_\infty$ is determined (i.e., $U_{\mathrm{ref}}/U_\infty = \pi/\sqrt{2} \approx 2.22$).

To examine Eqs. (8) and (9), we have conducted direct numerical simulations of the low-Reynolds-number flows over a flat plate at different AOAs at the Reynolds number $Re_c = U_\infty c/\nu = 200$. The flat plate with the straight corners is fixed in a uniform upstream flow, which has the chord length of $c = 1$ and the thickness of $0.01c$. The NS equations for an incompressible flow are solved on a Cartesian mesh with block structured adaptive mesh refinement, where the nonslip boundary condition on the surface of the flat plate is handled

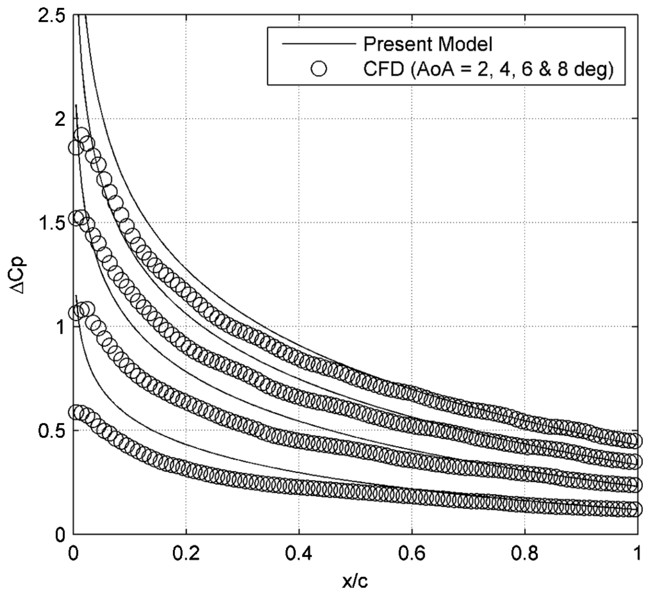

Fig. 2 Chordwise distributions of the surface pressure coefficient difference across the flat plate at different AOAs.

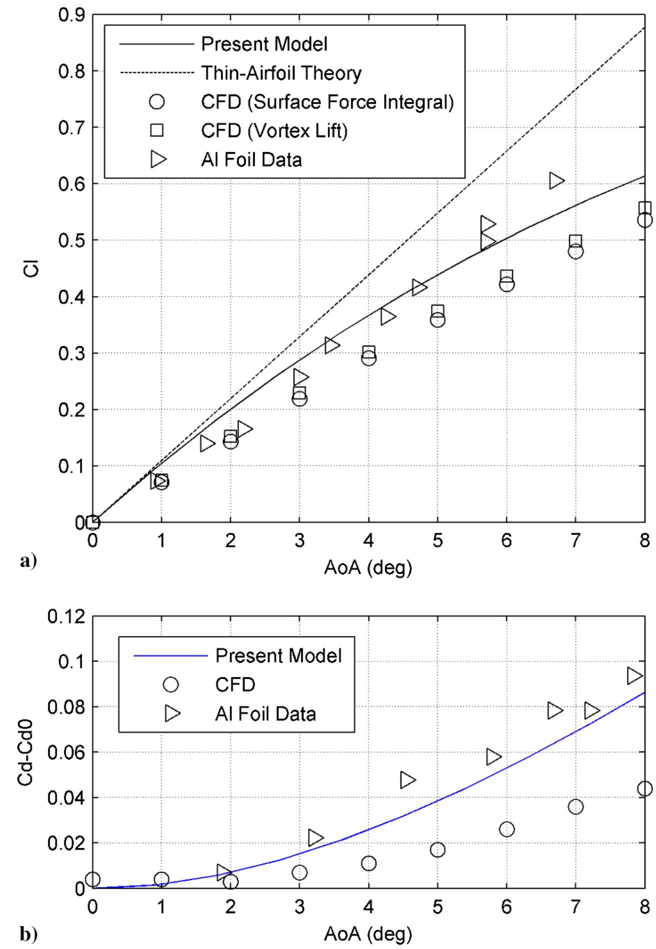

Fig. 4 Lift and drag coefficients as a function of AOA: a) lift, and b) drag.

by the immersed boundary method. The boundaries conditions on the other boundaries are the same as those described in [13,16]. The simulations are conducted in a domain of $[10c, 20c] \times [-10c, 10c]$, with a minimum grid length of $20/2048 \sim 0.01$. The numerical method and its validation are described in detail in [13,16].

Figure 1 shows a typical normalized vorticity field of the viscous flow over a flat plate at $Re_c = 200$. The ΔC_p distributions on the flat plate at different AOAs are shown in Fig. 2. It is found that the Kutta condition of zero pressure difference ($\Delta C_{p,\mathrm{TE}} = 0$) is not satisfied in this case. The value of $\Delta C_{p,\mathrm{TE}}$ depends on the AOA, which is given by an empirical formula extracted from the computational fluid dynamics (CFD) data (i.e., $\Delta C_{p,\mathrm{TE}} = a_1 + a_2\alpha + a_3\alpha^2$, with α in degrees), where $a_1 = -0.002599$, $a_2 = 0.06412$, and $a_3 = -0.000963$. When the parameters in Eq. (7) are $U_{\mathrm{ref}}/U_\infty = \pi/\sqrt{2}$ and $Re_c = 200$, as shown in Fig. 2, Eq. (7) gives the consistent profiles with the CFD data. Furthermore, Fig. 3 shows the profiles of $\Delta C_p/\alpha$ in comparison with

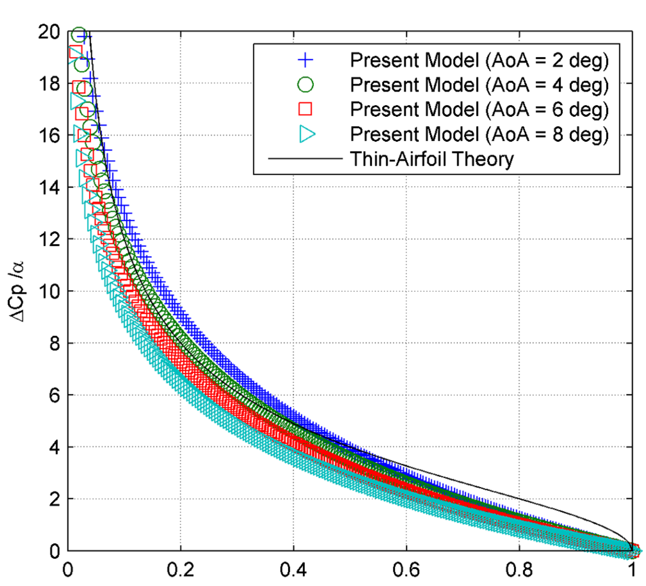

Fig. 3 Comparison between the present model and thin-airfoil theory in the normalized chordwise distributions of the surface pressure coefficient difference across the flat plate.

$\Delta C_p/\alpha = 4\sqrt{(1-\bar{x})/\bar{x}}$ given by the classical thin-airfoil theory, where the Kutta condition $\Delta C_{p,\mathrm{TE}} = 0$ is imposed for the purpose of comparison. Interestingly, all the profiles of $\Delta C_p/\alpha$ are approximately collapsed, indicating that the effect of α on ΔC_p predicted by the present model is essentially the same as that given by the thin-airfoil theory.

The lift coefficient in CFD is calculated by using both integration of the surface pressure and skin-friction fields and the vortex lift formula Eq. (11). Figure 4 shows the lift and drag coefficients (C_l and C_d) as a function of AOA. The C_l curve predicted by Eq. (8) exhibits the nonlinear behavior that is consistent with the CFD results and measurement data of an aluminum (Al) foil wing with the aspect ratio of 6 at the Reynolds number of 7500 [17], whereas the classical thin-airfoil theory gives the linear C_l curve with the larger lift slope. This example of the low-Reynolds-number flow indicates that the nonlinear effect of the viscosity on the lift generation could not be sufficiently simulated by the Kutta condition imposed in a simple linear potential-flow model. Furthermore, as shown in Fig. 4b, the C_d curve is in reasonable agreement with the data given by CFD and measurements [17], and $C_d - C_{d,0}$ increases with α mainly due to the pressure drag.

It is noted that the viscous diffusion effect could be incorporated into the kernel of the linear thin-airfoil integral equation, and thus the vortex-sheet strength distribution could be solved for predicting the lift coefficient of a thin airfoil at different Reynolds numbers [18]. The viscous diffusion effect mainly weakens the singularity of the kernel. This viscous thin-airfoil theory gives the normalized lift slope $(2\pi)^{-1} dC_l/d\alpha$ that varies from 0.9 to 1.3 depending on the Reynolds number and the geometric parameters of an airfoil. Nevertheless, the predicted C_l curve is still a linear function of α

because the fundamental integral equation has the same form as that in the classical thin-airfoil theory. Thus, this generalized theory could not include the nonlinear effect of the viscosity on the lift generation particularly at low Reynolds numbers.

IV. Vortex Lift

Furthermore, to elucidate the relationship between boundary layer and vortex lift, the viscous flow over a flat plate can be discussed from another perspective [14]. As shown in Fig. 1, the Lamb vector ($\boldsymbol{l} = \boldsymbol{u} \times \boldsymbol{\omega}$) that is the sole contributor to the lift in a steady attached viscous flow is concentrated in the boundary-layer domain V_{bl}, which forms a folded band with the width of δ and its outer contour C_{AB} wrapped around the plate from the point A to the point B in the wake. The vortex lift of a body in a viscous flow is expressed as a volume integral of the Lamb vector, i.e.,

$$L_{\mathrm{vor}} = \rho \boldsymbol{e}_l \cdot \int_{V_{bl}} \boldsymbol{l} \, \mathrm{d}V \tag{11}$$

where $\boldsymbol{e}_l$ is the unit vector normal to the freestream velocity. Formally, the vortex lift is expressed in the K-J theorem (i.e., $L_{\mathrm{vor}} = \rho U_\infty \Gamma_g$), where Γ_g is the generalized circulation based on the Lamb vector integral. Because the Lamb vector in a 2-D boundary layer is $\boldsymbol{u} \times \boldsymbol{\omega} = \boldsymbol{n} u\omega$, the lift calculated based on the pressure at the boundary-layer edge (on the contour C_{AB}) is related to the vortex lift by

$$L_{p_e} = \rho U_\infty \Gamma_g - \rho [\chi]_B^A \tag{12}$$

where $\chi = [u\omega]_0^\delta$ is the advective vorticity flux across the boundary layer, and $[\chi]_B^A$ is the jump of χ across the points A and B in the wake. Equation (12) indicates that, in the viscous flow, the pressure lift equals the vortex lift given by the K-J theorem only when the condition $[\chi]_B^A = [u\omega]_B^A = 0$ is satisfied in the wake. In other words, the positive and negative advective vorticity fluxes from the boundary layers on the upper and lower surfaces should be canceled out in the wake for the K-J theorem to be applicable in a viscous flow. This condition was first given by Taylor [19] to examine the applicability of the K-J theorem in viscous flows and elaborated by Sears for separated boundary layers [9,10]. Thus, it is referred to as the Taylor–Sears condition for the generation of the circulation [20].

For a very thin boundary layer, as the wake plane approaches the trailing edge, the Taylor–Sears condition would be reduced to the requirement that the pressures at the outer edges of boundary layers of the upper and lower surfaces must be the same at the trailing edge, which is the Kutta condition $\Delta C_p = 0$ at the trailing edge [10]. To examine the Taylor–Sears condition in this 2-D flow, we calculate the sum of the Lamb vector integrals across the boundary layers on the upper and lower surfaces (i.e., $\Delta l = [u\omega]_-^+$), where "+" and "−" denote the upper and lower surfaces, respectively. The Lamb vector difference Δl can be interpreted as the local loading on the flat plate according to Eq. (11), and at the same time $\Delta l = [u\omega]_-^+$ represents the net advective vorticity flux across the boundary layers on the flat plate. Figure 5 shows the chordwise distributions of $2\Delta l^*/\alpha = 2\Delta l/U_\infty^2\alpha$ on the flat plate at different AOAs in comparison with $\Delta C_p/\alpha$ given by the classical thin-airfoil theory. It is found that $\Delta l = [u\omega]_-^+ = 0$ at the trailing edge, and therefore the Taylor–Sears condition holds in this viscous flow even though the Kutta condition $\Delta C_p = 0$ is not satisfied at the trailing edge.

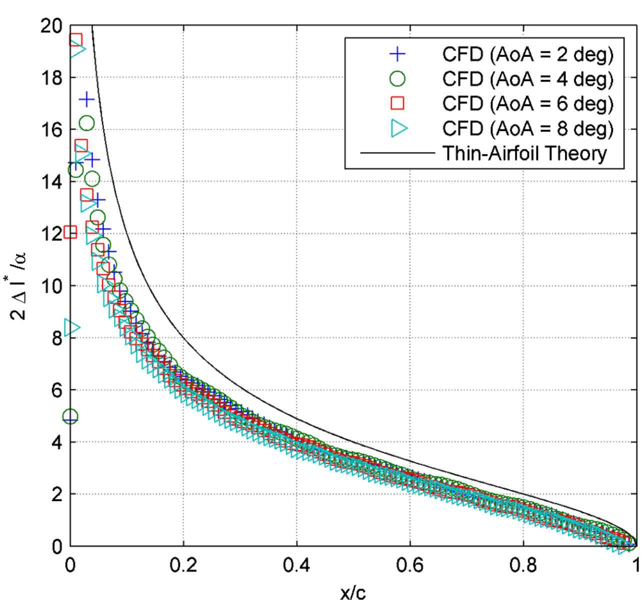

Fig. 5 Normalized chordwise distributions of the Lamb vector integral across the boundary layers on the flat plate.

V. Conclusions

The formal expression for the fluid-mechanical force (or aerodynamic force) of a body in an incompressible viscous flow is given, which explicitly elucidates the critical role of the fluid viscosity in generating the force (the lift particularly). The integrated pressure force is related to the accumulating effect of the viscous diffusion flux of the boundary vorticity magnitude along a skin-friction line. By using this force expression, the viscous flow over a flat plate at small angle of attack (AOA) is investigated in comparison with the numerical simulation and measurement data, and the nonlinear behavior of the lift coefficient as a function of AOA is predicted. Furthermore, the vortex lift is evaluated as the consequence of the viscous boundary layer, and the Taylor–Sears condition at the trailing edge in this viscous flow is examined.

References

[1] Prandtl, L., and Tietjen, O. G., *Fundamentals of Hydro- and Aeromechanics*, Dover, New York, 1934, Chap. 11.

[2] Milne-Thomson, L. M., *Theoretical Aerodynamics*, Dover, New York, 1958, Chap. 5.

[3] von Mises, R., *Theory of Flight*, Dover, New York, 1959, Chap. 8.

[4] Sears, W. R., *Introduction to Theoretical Aerodynamics and Hydrodynamics*, AIAA, Washington, D.C., 2011, Chap. 5.

[5] Batchelor, G., *An Introduction to Fluid Dynamics*, Cambridge Univ. Press, Cambridge, U.K., 1967, pp. 398–409.

[6] Stewartson, K., "D'Alembert's Paradox," *SIAM Review*, Vol. 23, No. 3, 1981, pp. 308–343.
doi:10.1137/1023063

[7] Crighton, D. G., "The Kutta Condition in Unsteady Flow," *Annual Review of Fluid Mechanics*, Vol. 17, 1985, pp. 411–445.
doi:10.1146/annurev.fl.17.010185.002211

[8] Glauert, H., *The Elements of Aerofoil and Airscrew Theory*, 2nd ed., Cambridge Univ. Press, Cambridge, England, U.K., 1983, Chap. 8.

[9] Sears, W. R., "Some Recent Developments in Airfoil Theory," *Journal of the Aeronautical Sciences*, Vol. 23, No. 5, 1956, pp. 490–499.
doi:10.2514/8.3588

[10] Sears, W. R., "Unsteady Motion of Airfoil with Boundary-Layer Separation," *AIAA Journal*, Vol. 14, No. 2, 1976, pp. 216–220.
doi:10.2514/3.7072

[11] Liu, T., Misaka, T., Asai, K., Obayashi, S., and Wu, J.-Z., "Feasibility of Skin-Friction Diagnostics Based on Surface Pressure Gradient Field," *Measurement Science and Technology*, Vol. 27, No. 12, 2016, Paper 125304.
doi:10.1088/0957-0233/27/12/125304

[12] Anderson, J. D., *A History of Aerodynamics*, Cambridge Univ. Press, Cambridge, England, U.K., 1997, Chap. 4.

[13] Wang, S. Z., Zhang, X., He, G. W., and Liu, T., "A Lift Formula Applied to Low-Reynolds-Number Unsteady Flows," *Physics of Fluids*, Vol. 25, No. 9, 2013, Paper 093605.
doi:10.1063/1.4821520

[14] Liu, T., Wang, S., Zhang, X., and He, G., "Unsteady Thin Airfoil Theory Revisited: Application of a Simple Lift Formula," *AIAA Journal*, Vol. 53, No. 6, 2015, pp. 1492–1502.
doi:10.2514/1.J053439

[15] Schlichting, H., *Boundary-Layer Theory*, 7th ed., McGraw–Hill, New York, 1979, Chaps. 7, 16.
[16] Wang, S., He, G., Zhang, X., and Liu, T., "Evaluation of Lift Formulas Applied to Low Reynolds Number Flows," *AIAA Journal*, Vol. 53, No. 1, 2015, pp. 161–175. doi:10.2514/1.J053042
[17] Okamoto, M., "An Experimental Study on Aerodynamic Characteristics of Steady and Unsteady Airfoils at Low Reynolds Number," Ph.D. Thesis, Univ. of Tokyo, Tokyo, Dec. 2006.
[18] Yates, J. E., "Viscous Thin-Airfoil Theory," Aeronautical Research Associate of Princeton Rept. 413, Princeton, NJ, Feb. 1980.
[19] Taylor, G. I., "Note on the Connection Between the Lift in an Airfoil in a Wind and the Circulation Round It," *Philosophical Transactions of the Royal Society of London A*, Vol. 225, 1926, pp. 238–245.
[20] Zhu, J. Y., Liu, T. S., Liu, L. Q., Zou, S. F., and Wu, J. Z., "Causal Mechanism in Airfoil-Circulation Formation," *Physics of Fluids*, Vol. 27, No. 12, 2015, Paper 123601. doi:10.1063/1.4937348

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