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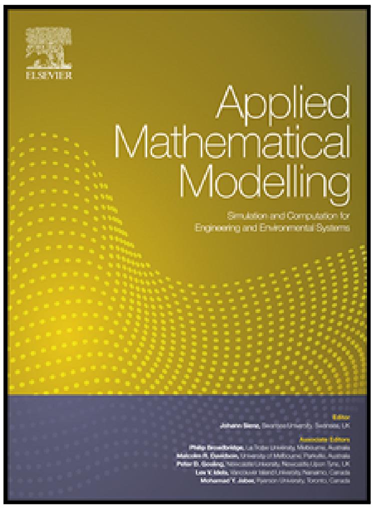

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Highlights

- The reliability-based design optimization and safety factor method is compared for the optimization result and process.
- The reliability-based design optimization method including probabilistic reliability and nonprobabilistic reliability are given.
- The viewpoint is illustrated by two numerical examples including a two-bar truss and a supersonic wing structure.

Comparison of the Reliability-based and Safety Factor Methods for Structural Design

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Abstract: Reliability-based design optimization theory has been widely acknowledged as an advanced and advantageous methodology for complex structural system design. Comparatively, the traditional safety factor design method has fallen out of favor with designers since it is simply just a comprehensive expression of all the uncertainties existing in a practical engineering structure, which has been verified to be unreasonable. Moreover, there is no description of the method for searching for the optimal design. In this paper, a comparison of the two **approaches** is performed for the optimization results and process. It is demonstrated that the weight of the designed structure by the reliability-based optimization is not heavier than that by the safety factor design method under the same reliability requirements. The time efficiency, error estimates and sensitivity analysis are also compared to discuss the advantages and disadvantages of the two **approaches**. Eventually, the advancement of the reliability-based design optimization is illustrated with optimization designs of a two-bar truss and one practical supersonic wing.

Keywords: Safety factor method; Reliability-based design optimization; Structural design; Uncertainty.

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1. Introduction

With the continuous development of technology, the complexity of engineered structural systems has gradually increased, meaning that the anticipated influence of uncertainty on these structures has become increasingly evident [1]. Currently, the value of the safety factor (SF) is normally confirmed as 1.5, with the SF method consistently dominating the industry [2] as a deterministic design method with limited improvement in providing efficient operational structures. All the inherent uncertain factors lead to large variations in the structural optimization models and hinder the development of the structural design. For the sake of considering the influence of the refined dispersion factors and satisfying the elaborate design requirements of modern structures, reliability-based design optimization (RBDO) methods, such as the probabilistic reliability-based design optimization (PRBDO) approach [3], fuzzy reliability [4] and nonprobabilistic reliability-based design optimization (NRBDO) method [5], have been proposed to increase the structural stock utilization and solve refinement design problems.

The SF method assumes that the material strength and the load-induced stress are deterministic values and that their ratio is an arbitrarily specified ultimate SF to account for design uncertainties. **Wang et al. [6]** made efforts to find a new approach by combining the SF and the reliability concept for structural design, such as in reference. The SF method is universally applicable to most structural problems and has been verified by **Verderaime [7]**. However, this method is usually too conservative, which is deficient for the practical application of structural engineering.

The concept of probabilistic reliability is established on the basis the application of probability theory and mathematical statistics in structural safety measure. Cornell [8] put forward the **first-order reliability method** to evaluate the degree of structural safety with a reliability index. Hasofer et al.[9] suggested calculating the reliability index according to the failure surface rather than the function definition to solve the inconsistency of the reliability index, which is referred to as the **advanced first-order reliability method.** For the sake of calculating the failure probability with a nonlinear limit state function and nonnormal random variables, Rackwitz et al. [10] proposed the R-F algorithm, which has been adopted by most scholars as a recognized method. Meanwhile, Breitung [11] utilized the s**econd-order reliability method** to calculate reliability indexes with strong nonlinearity.

The PRBDO method is the most mature theory of the current uncertainty analysis, and has been used in the academic and engineering fields [12]. Meanwhile, the NRBDO models, such as the interval set model [13, 14] and convex model [15, 16], have been suggested for addressing nondeterministic structural analysis and optimization problems with limited uncertain source information [17]. Ben-Haim et al. [18] first introduced the concept of nonprobabilistic reliability through convex model theory. Then, Qiu et al. [19] and Wang et al. [20] developed a structural optimization method based on uncertain-but-bounded parameters. Among these two methods, the PRBDO method treats the uncertainties as random variables with assumed probability distributions, while the NRBDO method constructs the uncertainties as an interval set model [21] and a convex model [22].

Some scholars have studied the similarities and differences between **the RBDO and SF methods. Elishakoff [2]** indicated that the SF is closely related to the reliability of structures. In addition, many scholars have combined reliability and the SF to search for more efficient optimization strategies. Ching [23] investigated the interrelations between the SF and the nonprobabilistic reliability, establishing the functions of the SF and the nonprobabilistic reliability measures. However, most studies adopt a traditional or central SF design and then evaluate the reliability of the corresponding schemes [24, 25]. Differing from the central safety factor design method, in the reliability-based safety factor design method, the safety factor is defined in such way that the probability of the ratio of the allowable over response that is less than the safety factor is equal to the allowable probability [26]. It is noted that the design space of this kind of SF method is still not as large as the RBDO design space.

Above all, the existing literature has no analysis of the pros and cons of the two methods, and there are no essential accounts for the conservativeness of conventional SF methods. This study focuses on why the RBDO method can obtain better structural optimal results. The optimal process and results of **the RBDO (including the PRBDO and NRBDO) and SF methods** are compared. In this paper, the two methods are first elaborated on. Then, the comparison between the two methods from the optimization results and the optimization process is described.

2. Safety factor method for aircraft structure

2.1 Definition of the safety factor

In the recent years, the SF method has been widely applied and developed due to its convenience and conservation for the performance of aircraft. The civil aviation regulations of the United States describes the SF as follows:

(1) The allowable load is the maximum load at predetermined and normal conditions;

(2) The limit load equals the allowable load multiplied by an appropriate SF;

(3) The SF is a design factor that is used to prevent the possibility of a working load greater than the allowable load due to the inaccuracy of the design.

In the early stage of SF method for aircraft structure, different countries adopted diverse values, e.g. in 1928, the USA adopted 2.0 as the SF; Germany adopted 1.8 as the SF; the UK adopted 1.15 as the SF. The values of SF changed due to the engineering practice of the aircraft structural design. Afterwards, all countries adopted 1.5 as the SF of aircraft structural design, because the ratio of ultimate strength and elastic limit of aluminum alloy approximately is 1.5. The development of SF for aircraft structure was reviewed in Refs. [2, 27, 28].

Up to now, many aircrafts have been designed based on the SF theory and many scholars have researched the quantification of the SF [7, 24, 25]. It should be noted that the selection of the SF is affected by many factors in the structural design of the aircraft. Scholars believe that the main factors can be divided into four aspects: (a) the accuracy of the calculated load; (b) the use environment; (c) the material's qualitative and craft factors; and (d) the requirements of plastic deformation and stiffness.

2.2 Design model of the SF method

The SF method is a design method, in which the constraint condition is the strength requirement related to the SF as in the optimization model, as shown below:

$$\begin{cases} \text{find} & \boldsymbol{X}=[x_1,x_2,\cdots,x_n] \\ \min & M(\boldsymbol{X}) \\ \text{s.t.} & q=\dfrac{h(\boldsymbol{P}^c)}{n_0}-g(\boldsymbol{X},\boldsymbol{P}^c)\geq 0 \\ & x_i\in\left[\underline{x_i},\overline{x_i}\right],\ i=1,2,\cdots,n \end{cases} \tag{1}$$

where $\boldsymbol{X}=[x_1,x_2,\ldots,x_n]$ are the design variables, $\boldsymbol{P}=[p_1,p_2,\ldots,p_m]$ represents the uncertain interval parameters involved in the analysis process, $\boldsymbol{P}^c$ indicates the mean value or middle value of the uncertain parameters; $M(\boldsymbol{X})$ denotes the structural weight of the design variables; $q=\frac{h(\boldsymbol{P}^c)}{n_0}-g(\boldsymbol{X},\boldsymbol{P}^c)$ represents the limit state function; $q \geq 0$ indicates that the structure is safe and $q<0$ indicates that the structure is not safe; $h(\bullet)$ is the allowable limit state function, which has nothing to do with the design variables; $g(\bullet)$ denotes the calculated response function; and n_0 indicates the safety coefficient in the design of the structure. For convenience, the optimal design scheme obtained by the SF method is defined as $\boldsymbol{X}_1^*$.

3. Reliability-based design optimization theory

Consider the dispersion of structural parameters $\boldsymbol{P}$, such as the loads and material strengths. The reliability η_1^* of the optimal design scheme $\boldsymbol{X}_1^*$ can be expressed as follows:

$$\eta_1^*=\eta(h(\boldsymbol{P}^c,\Delta\boldsymbol{P})-g(\boldsymbol{X}_1^*,\boldsymbol{P}^c,\Delta\boldsymbol{P})\geq 0) \tag{2}$$

where $\Delta\boldsymbol{P}$ represents the radius of the interval parameters. The object of finding the structural reliability of $\boldsymbol{X}_1^*$ is to provide the constraint conditions for the subsequent RBDO design.

The RBDO method is established on the basis of reliability theory. The uncertain structural responses are obtained through an uncertainty propagation analysis, e.g., the vertex theory [29], the series expansion method [30], the stochastic process [31] and the collocation method [32]. Then, the reliability index is established by the intersection relationship between the response and the allowable values. Under normal circumstances, the structural design model of the RBDO method can be represented as follows:

$$\begin{cases} \text{find} & \boldsymbol{X}=[x_1,x_2,\cdots,x_n] \\ \min & M(\boldsymbol{X}) \\ \text{s.t.} & \eta(h(\boldsymbol{P}^c,\Delta\boldsymbol{P})-g(\boldsymbol{X},\boldsymbol{P}^c,\Delta\boldsymbol{P})\geq 0)\geq \eta_1^* \\ & x_i\in\left[\underline{x_i},\overline{x_i}\right],\ i=1,2,\cdots,n \end{cases} \tag{3}$$

where η_1^* denotes the structural reliability obtained from the conventional SF method, which will be the optimization design constraints of the RBDO method; $h(\bullet)$ is the allowable limit state function; $g(\bullet)$ denotes the calculated response function; $\boldsymbol{P}^c$, $\Delta\boldsymbol{P}$ expresses the mean and deviation of the uncertainties, respectively; and $\eta(h(\boldsymbol{P}^c, \Delta\boldsymbol{P}) - g(\boldsymbol{X}, \boldsymbol{P}^c, \Delta\boldsymbol{P}) \geq 0)$ indicates structural reliability under the influence of the considered load and material dispersion. Because RBDO is based on the optimal results of the conventional SF method, the RBDO results are therefore not going to be worse than the SF design results under the same reliability.

The optimal design point $\boldsymbol{X}_2^*$ can be captured through the RBDO method, and the corresponding reliability η_2^* can be represented as follows:

$$\eta_2^* = \eta(h(\boldsymbol{P}^c, \Delta\boldsymbol{P}) - g(\boldsymbol{X}_2^*, \boldsymbol{P}^c, \Delta\boldsymbol{P}) \geq 0) \tag{4}$$

The reliability design method mainly includes the PRBDO and the NRBDO methods according to the type of uncertainties. Both of these methods are employed in this paper for comparison with the SF design optimization method.

3.1 Probability reliability-based design optimization method

In the analysis of structure safety, the safety degree can be evaluated by virtue of the limit state function, which is usually represented as follows:

$$q(\boldsymbol{X}) = q(X_1, X_2, \ldots, X_m) \tag{5}$$

where $q(\boldsymbol{X})$ is the limit state function of the structure, with $q(\boldsymbol{X}) < 0$ defining the failure events, and $\boldsymbol{X} = [X_1, X_2, \cdots, X_m]^T$ denotes the vector of the random variable in the structural design.

Assuming that $f_x(\boldsymbol{X})$ is the joint probability density function of the random vector $\boldsymbol{X}$, the structural reliability η is expressed as:

$$\eta = P(g(\boldsymbol{X}) \geq 0) = \int_{q(\boldsymbol{X}) \geq 0} f_x(\boldsymbol{X}) dx_1 dx_2 \cdots dx_m \tag{6}$$

where $P(\cdot)$ represents the probability for the degree of reliability.

There are currently two common methods for the integral solution, which are the numerical **simulation and the approximate method**. Although the numerical simulation method can provide a higher accuracy, the large amount of calculation effort required makes it too difficult for application in a practical engineering structure. By virtue of the response surface method, which is used to approximate the real failure plane, the structural reliability index can be obtained by **the first-order reliability and second-order reliability methods**.

In general, the **first-order reliability method** transforms the random variables into the space of independent standard normal variables; correspondingly, the failure plane $q(\boldsymbol{x})=0$ can be converted to a standardized failure plane $Q(\boldsymbol{u})=0$. Eventually, the structural reliability η can be acquired through the reliability index β, where the geometric meaning of the reliability index β is the shortest distance between the point of origin and the failure plane in standardized space $\boldsymbol{u}$. In conclusion, the structural optimization model can be demonstrated as follows:

$$\begin{cases} \text{find} & \boldsymbol{u} \\ \min & \beta=\sqrt{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}} \\ \text{s.t.} & Q(\boldsymbol{u})=0 \end{cases} \tag{7}$$

As for the linear limit state function indicated below:

$$q(\boldsymbol{X})=a_0+a_1X_1+a_2X_2+\cdots+a_mX_m \tag{8}$$

where $a_i, i=1,2,\cdots,m$ is an arbitrary constant, the reliability index can be obtained using first-order reliability method:

$$\beta=\frac{\mu_M}{\sigma_M}=\frac{a_0+\sum_{i=1}^{m}a_i\mu_{X_i}}{\sqrt{\sum_{i=1}^{m}a_i^2\sigma_{X_i}^2}} \tag{9}$$

where μ_{X_i} and σ_{X_i} denote the expectation and standard deviation of the normal random variable, respectively. Then, the structural reliability can be obtained through the formula shown below:

$$\eta=\Phi(\beta) \tag{10}$$

where $\Phi(\cdot)$ represents the standard normal distribution function.

It should be noted that first-order reliability method executes its sensitivity analysis relatively more easily and with less calculations compared to second-order reliability method. However, a large error may exist when the design point has a strong nonlinearity.

3.2 Nonprobability reliability-based design optimization method

The interval model and convex set-theoretic are the primary models of the NRBDO method, which is expounded below mainly on the basis of the interval model in this article.

The safety analysis of a structure subject to external loads is considered. The stress S and strength R are influenced by a great number of uncertainties, and thus they can be expressed as the following functions of variables:

$$\begin{aligned} S^I &= S(p_1^I, p_2^I, \ldots, p_m^I) = [\underline{S}, \overline{S}] \\ R^I &= R(p_1^I, p_2^I, \ldots, p_m^I) = [\underline{R}, \overline{R}] \end{aligned} \tag{11}$$

where $p_i^I\,(i=1,2,\ldots,m)$ are the uncertain variables that are related to the stress and strength, such as force, moment, temperature, surface roughness, material properties, etc. S^I and R^I are the interval representations of stress and strength, respectively; $\overline{S}$ and $\underline{S}$ are the upper and lower bounds of the structural stress, respectively, which can be acquired by means of an interval propagation analysis; $\overline{R}$ and $\underline{R}$ are the upper and lower bounds of the structural strength respectively.

Once the stress interval S^I and the strength interval R^I are determined, the nonprobabilistic reliability index can be computed based on the nonprobabilistic set-theoretic model proposed by Wang et al. [33]. For the following convenience, the nonprobabilistic set-theoretic model of the structural safety measure will simply be introduced here.

The limit state function of the structure is expressed as a function of the stress S and the strength R R as follows:

$$q(R,S) = R - S \tag{12}$$

It is acknowledged that the structure is safe when $q(R,S) > 0$; on the contrary, the structure is in a state of failure when $q(R,S) < 0$. As a result, $q(R,S) = 0$ denotes the limit state plane of the structure, which divides the basic variables space into two parts, namely, the safe region and the failure region, as shown in Fig. 1.

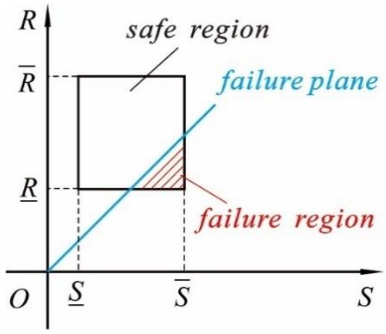


Fig. 1. Nonprobabilistic set-theoretic stress–strength interference model.

It is reasonable to assume that the structure is safe when the stress is less than the strength, but interference between the stress and strength will occur because of the dispersion of uncertainty factors, which is referred to as stress–strength interference model.

When interference occurs between the stress and strength interval variables, the measure of the reliability index is based on the interval interference model and the concept of the volume ratio, which means the safety level of the structure is defined as the ratio of the safe region volume to the total volume. For a two-dimensional scenario, the measurement degenerates into the ratio of the safe region area S_{safe} to the total area S_{total}, as shown in Fig. 1.

According to the definition of nonprobabilistic reliability, there are six different formula forms for calculating the reliability, which are separated by the size relation between the stress and strength interval boundary. The formula for solving the nonprobabilistic reliability is demonstrated as follows:

$$\eta = \begin{cases} 1 & ,\underline{R} > \overline{S} \\ 1 - \dfrac{\left(\overline{S} - \underline{R}\right)^2}{8R^r S^r} & ,\underline{S} \le \underline{R} \le \overline{S} \le \overline{R} \\ \dfrac{S^r - S^c + R^c}{2S^r} & ,\underline{S} \le \underline{R} \le \overline{R} \le \overline{S} \\ \dfrac{R^r + R^c - S^c}{2R^r} & ,\underline{R} \le \underline{S} \le \overline{S} \le \overline{R} \\ \dfrac{\left(\overline{R} - \underline{S}\right)^2}{8R^r S^r} & ,\underline{R} \le \underline{S} \le \overline{R} \le \overline{S} \\ 0 & ,\overline{R} < \underline{S} \end{cases} \tag{13}$$

For the sake of convenience, the stress and strength interval variables can be normalized and standardized as follows:

$$\delta_S = (S - S^c)/S^r,\quad \delta_R = (R - R^c)/R^r \tag{14}$$

where $S^c = (\bar{S} + \underline{S})/2$ and $S^r = (\bar{S} - \underline{S})/2$ represents the central value and radius of the interval variable S^I, respectively; $R^c = (\bar{R} + \underline{R})/2$ and $R^r = (\bar{R} - \underline{R})/2$ denotes the central value and radius of the interval variable R^I, respectively. Meanwhile, δ_S and δ_R are standardized interval variables. Substituting Eq. (14) in Eq. (12) gives:

$$q(\delta_R, \delta_S) = R^r \delta_R - S^r \delta_S + (R^c - S^c) \tag{15}$$

where $q(\delta_R, \delta_S)$ is the failure plane in the space of the standardized variables as shown in Fig. 2.

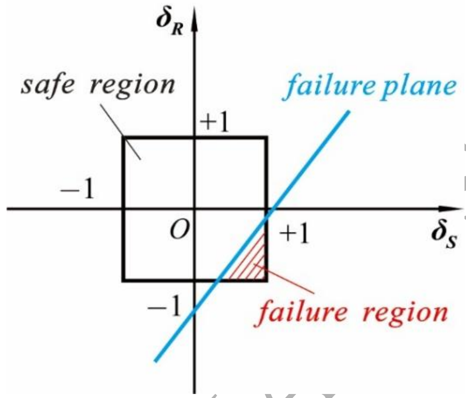


Fig. 2. Space of the standardized variables.

All in all, the PRBDO method can be employed to optimize the structural design when the probability density function is known, but statistical information on the uncertainty may not be easily available, whereas the bounds on the uncertain information can be readily obtained. Under this circumstance, the NRBDO method would be an alternative choice for designers. The flowchart of the RBDO method is shown in Fig. 3. It is noted that the RBDO method can guarantee that the optimal structure does not have worse reliability than the optimal structure obtained by the SF method. It can also be seen that the SF-based optimization and reliability analysis is the basis of the RBDO method.

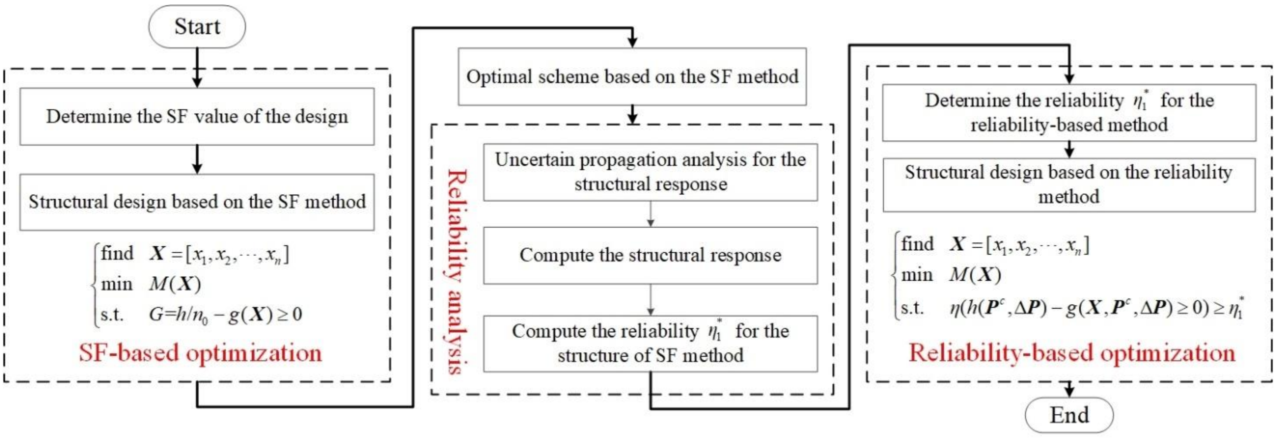


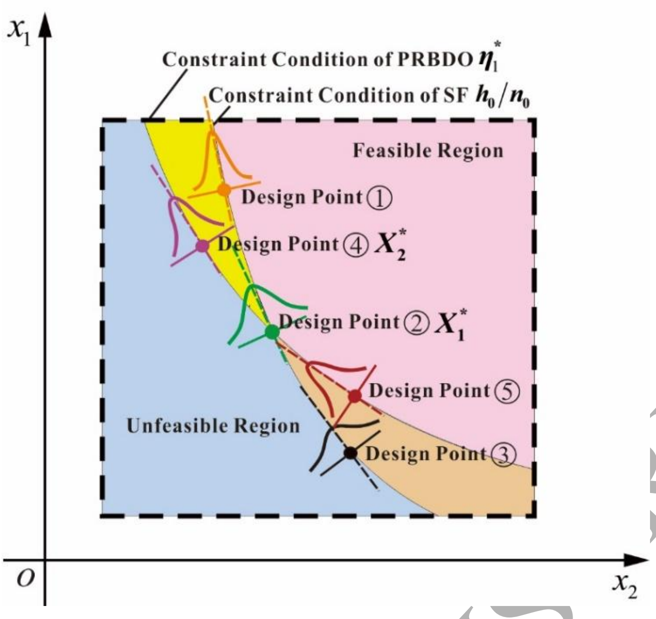
Fig. 3 Flowchart of the RBDO method.

4. Comparison of the RBDO and SF methods

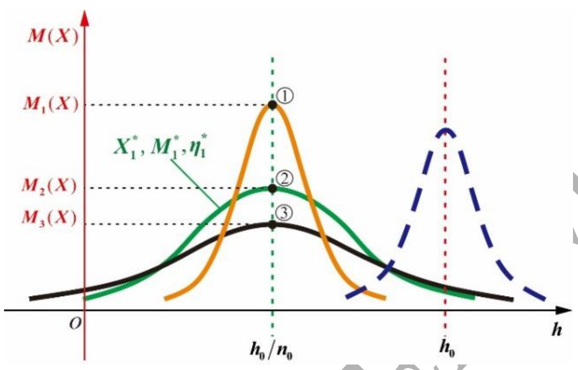
In this section, **the RBDO and SF methods** are compared from two aspects, namely, the optimization solution and the optimization process. In the comparison of the optimization results, the searching process of **the RBDO (including the PRBDO and the NRBDO methods) and SF methods** are given and compared. In the comparison of the optimization process, the time efficiency, accuracy and sensitivity analysis of the two methods are compared.

4.1 Comparisons of the optimization results

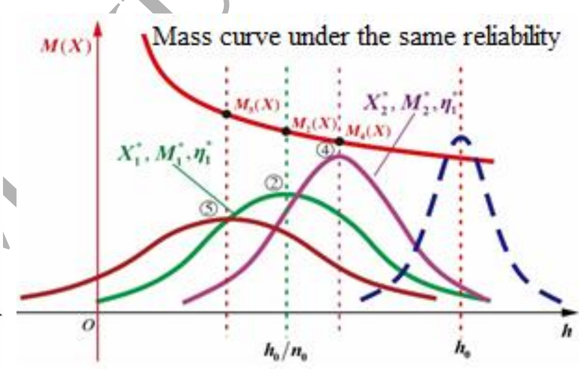
For the central SF design method, the design process simply considered the mean values of the uncertain parameters rather than taking the variance effect into account. As a result, the design result may only appear on the line of a constant value h_0 / n_0. As shown in Fig. 4(b), the green-dashed line represents the structural response of the SF method and the red-dashed line indicates the structure's allowable value, while the X coordinate denotes the structural response value and the Y coordinate indicates the design objective of the structure. In this mathematical model, the X coordinate is the structural stress and the Y coordinate represents the mass of the structural system. Obviously, the structural mass and stress present an inversely proportional relationship, which means the structural mass will decrease when the stress in the structure increases, just as the red-solid line demonstrates in Fig. 4(c). The structural stress value will be as far from the maximum as possible under constraint conditions, while the design point will continuously search from the top to the bottom in the green-dashed line until reaching the minimum mass limit, namely, the optimal design point. In Fig. 4(b), the intersection point ② is the optimal design results of the SF method (point ① is a conservative result while point ③ is dissatisfactory due to the constraint conditions).



(a)



(b)

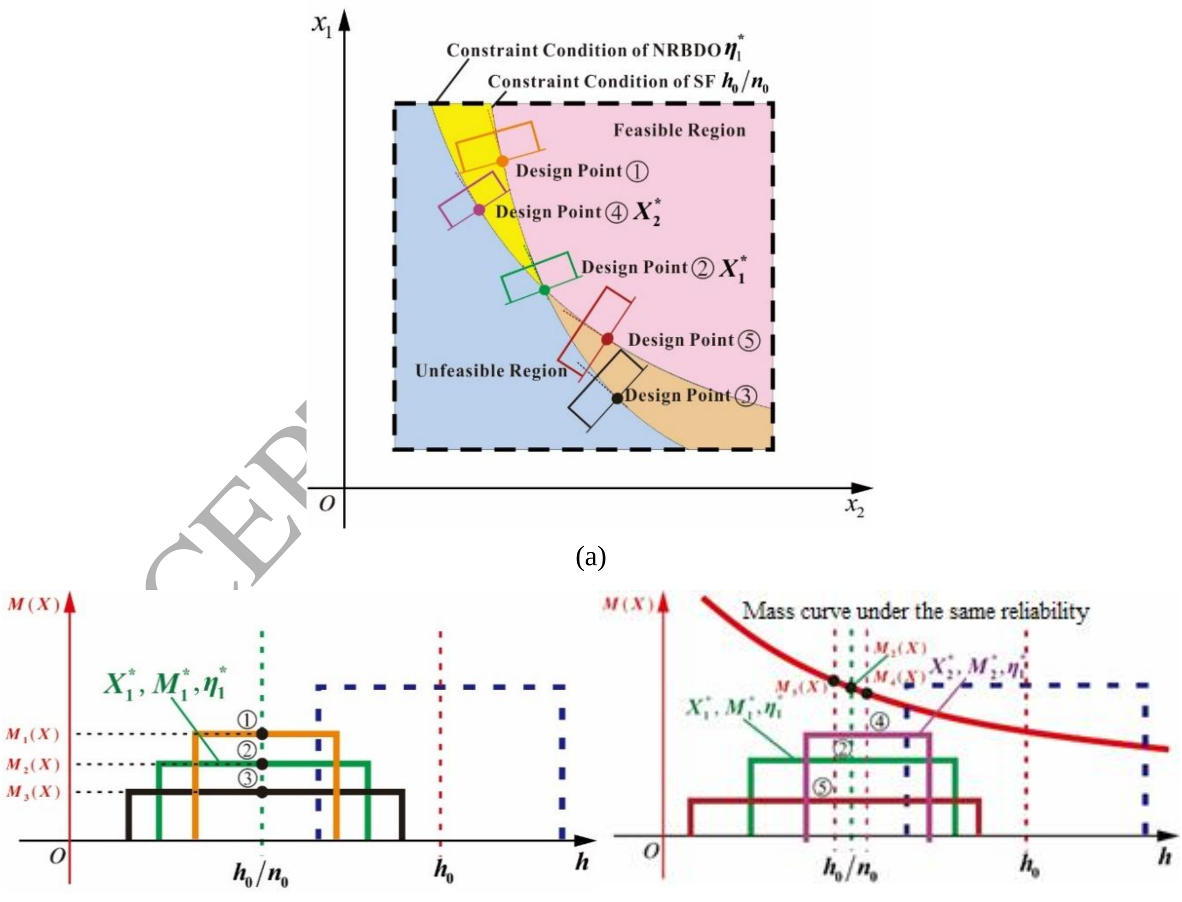


(c)

Fig. 4. Comparison of the optimization process between the PRBDO and SF methods: (a) Feasible region of the two methods, (b) Searching process of the optimal design point of the SF method, (c) Searching process of the optimal design point of the PRBDO method.

While it is known that the traditional SF method is a deterministic design because of the general consideration of uncertainties, the safety degree of the structure not only depends on the influence of the mean value but is also affected by the variances of the parameters existing in the practical engineering structure. Therefore, the feasible solutions of the SF design model do not always meet the reliability constraints due to the dispersion coming from the structural parameters, which means the concepts and the design results of the SF method are unreasonable and defective to the structural safety.

With the development of design philosophy and science, the PRBDO method was introduced into the design of the structure, which considered the impact of the mean value and variance during the design process. As a result of considering the influence of uncertain factors, other feasible design regions may exist due to the same structural reliability η_1^* coming from the SF method, which is treated as a constraint condition in the PRBDO method. As shown in Fig. 4 (a), **the PRBDO and SF methods** have different design domains, so it is more likely that other design schemes exist where the reliability of the structure is satisfied and the structural response is different from h_0 / n_0 while being simultaneously born in the SF method. Because of the consideration of the variance effect, the structural response values of the RBDO method would be different from that of the SF method. In other words, the design point of the RBDO can move around the green-dashed line and obtain a global optimal solution in the case of satisfying the constraint conditions. As shown in Fig. 4 (c), the intersection point ④ is the optimal design results of the NRBDO method while point ⑤ is dissatisfactory due to the constraint conditions. Therefore, it is reasonable that the results by the RBDO method are better than the results by the SF method.



(a)

(b) (c)

Fig. 5. Comparison of the optimization process between the NRBDO and SF methods: (a) Feasible regions of the two methods, (b) Searching process of the optimal design point of the SF method, (c) Searching process of the optimal design point of the NRBDO method.

Similarly, as demonstrated in Fig. 5, the deterministic SF method can only search for the optimal design point based on the green-dashed line, while the NRBDO method can look for the optimal design solution within the scope of the global domain. Thus, the NRBDO method is easier to obtain the global optimal design solution.

In view of the previous section, we know that the results of the RBDO method are no worse than the conventional SF method, but sometimes situations exist where these two methods obtain the same optimal design points, so in what circumstances are the results of the RBDO method better than SF method? In the following paragraphs, the conditions when the results of the RBDO method are better than the SF method will be demonstrated.

Before the deduction and discussion, we must obtain the following consensus:

(1) In the design of the structural system, the mass of the structure and the structural response (such as the stress and displacement) present the relationship of the inverse proportion, just as the red-solid line shows in Fig. 4 (c);

(2) As for the uncertain interval response parameters S_1 and S_2, when $S_1^r \leq S_2^r$, the reliability of the two methods can be the same only if $S_1^r \leq S_2^r$ (the details are shown in Sec 4.1.2);

(3) Because of the consideration of the dispersion of parameters in the beginning of the structural design, the structural response values of the RBDO method can move around the mean value results to find the global optimal design scheme

Based on the three consensuses above, we can make an analysis of circumstance when the results of the RBDO can reduce the structural weight on the basis of the results of the SF method under the same constraint of reliability.

When the structural response of the RBDO method appears on the left side of the central value (the central value is the result of the conventional SF method), the optimal structural mass is increased, although it can still meet the reliability constraints, which means a scenario with a decreasing structural response value cannot satisfy the conditions.

When the structural response of the RBDO method appears on the right side of the central value, only the second consensus is satisfied, so the structural mass can be decreased under the reliability constraints. Namely, the RBDO method can reduce the structural weight compared with the SF method when the variance of the structural response is decreased, otherwise, these two methods will result in the same optimal scheme.

In this following section, the PRBDO and NRBDO methods are compared with the SF method to analysis the weight reduction effect under the same reliability constraints.

4.1.1 Comparison between the PRBDO and SF methods

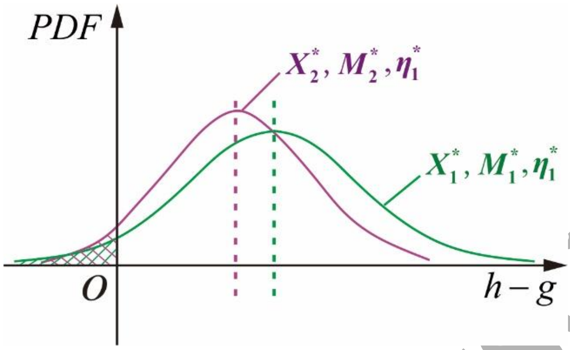


Fig. 6. PDF of the limit state functions based on the RBDO and SF methods.

The probability density function (PDF) of the limit state functions based on the RBDO and SF methods are displayed in Fig. 6. The green line denotes the PDF of the limit state function for the SF method, while the purple line indicates the PDF of the limit state function for the RBDO method. The structural reliability is calculated for the optimal structure based on the SF method as follows:

$$\eta_1 = \Phi(\frac{\mu_M(\boldsymbol{X}_1)}{\sigma_M(\boldsymbol{X}_1, \boldsymbol{P})}) \tag{16}$$

The structural reliability of the RBDO is represented as follows:

$$\eta_2 = \Phi(\frac{\mu_M(\boldsymbol{u}_2)}{\sigma_M(\boldsymbol{X}, \boldsymbol{u}_2)}) \tag{17}$$

The constraint condition is $\eta_2 \geq \eta_1$ in the reliability optimization. Meanwhile, the function $\Phi(\cdot)$ is monotonous. Thus, the following inequality could be given.

$$\frac{\mu_M(\boldsymbol{u}_2)}{\sigma_M(\boldsymbol{X}, \boldsymbol{u}_2)} \geq \frac{\mu_M(\boldsymbol{u}_1)}{\sigma_M(\boldsymbol{X}, \boldsymbol{u}_1)} \tag{18}$$

Actually, $\mu_M(\boldsymbol{u})$ is positively correlated with $\boldsymbol{u}$, because the more mass the structure has, the larger the design space is and the higher the structural safety margin is. Herein, the design variable $\boldsymbol{u}_1$ is positively correlated with the structural mass. To ensure $\mu_M(\boldsymbol{u}_2) \le \mu_M(\boldsymbol{u}_1)$, $\frac{\sigma_M(\boldsymbol{X},\boldsymbol{u}_2)}{\sigma_M(\boldsymbol{X},\boldsymbol{u}_1)} \le \frac{\mu_M(\boldsymbol{u}_2)}{\mu_M(\boldsymbol{u}_1)} \le 1$. In other words, the standard deviation of the limit state function for the optimal scheme of the PRBDO method should be larger than the SF method.

Above all, the PRBDO method has the chance of reducing the mass compared with the design scheme based on the SF method by reducing the deviation.

4.1.2 Comparison between the NRBDO and SF methods

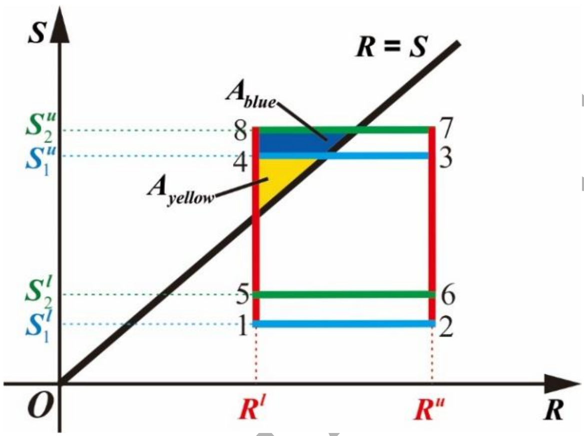


Fig. 7. **Two-dimensional interval analysis model.**

In the analysis with two dimensions, we marked the total area as A. The yellow area is defined as A_{yellow}, which represents the failure probability of the SF method, and the blue area is the incremental area because of the change of the response value, where the initial reliability of the structure is defined through the volume ratio proposed by Wang [33]. This can be expressed as follows:

$$\eta_1 = 1 - \frac{A_{yellow}}{A} \tag{19}$$

Assuming that the variances of the structural response and total area remain the same when the mean value is increased, the reliability can be given:

$$\eta_2 = 1 - \frac{A_{blue} + A_{yellow}}{A} \tag{20}$$

The total area must be increased if the constrain condition $\eta_1 \le \eta_2$ need to be satisfied, which violates the previous assumptions. Thus, the assumption fails.

When the variance of the structural response is increased along with the mean value, the area ratio of the shadow becomes larger. As a result, the failure probability is increased and the reliability is decreased, which dissatisfies the requirements of the reliability constraint.

Only in the situation where the mean value is increased while the variance of the structural response is decreased will the failure probability decrease and the reliability increase, which is the desired result in the structural design. Therefore, only in the case where the mean value is increased while the variance of the structural response is decreased is the result of the RBDO method better than the SF method. Otherwise, the optimal results of the two methods will have the same optimal scheme.

4.2 Comparison of the optimization process

To compare the application of the two methods in the structural design, the time efficiency, error estimates and sensitivity analysis are discussed.

4.2.1 Time efficiency

The time efficiency is relevant to the iteration time and the **number of deterministic structural analysis** in each iteration. **The deterministic structural analysis represents the calculation of structural responses when the system parameters and loads are deterministic.** The iteration time is a function of the optimization model, optimization algorithm, the gradient at the optimal point, etc. For different structural optimizations, the optimization models of **the RBDO and SF methods** are different. Thus, the comparison of the iteration time of the two methods has no uniform conclusion. Therefore, the number of deterministic structural analysis in each iteration is measured to compare the efficiency. In every iteration, the SF method only performs one analysis for the optimization. However, the RBDO method needs more than one analysis for each iteration, which is dependent on the type of uncertainties and the uncertainty propagation analysis method. For different uncertainty propagation methods, the number of deterministic structural analysis for the structural response g are listed in Table 1, in which m represents the number of uncertainties. From the table, one can see that all of the RBDO methods need no fewer than 2 analysis in every optimal iteration. Overall, the RBDO methods has a worse calculation efficiency compared with the SF methods.

Table 1 Number of deterministic structural analysis for different RBDO methods

Type of reliability	Propagation method	Number of deterministic structural analysis
PRBDO	Series expansion [34]	$\geq m+1$
	Stochastic process [35]	$\geq 10^{m}$
NRBDO	Vertex theorem [36]	2^{m}
	Series expansion [30]	$\geq m+1$
	Collocation theorem [32]	$\geq 3m$

4.2.2 Accuracy

The SF method evaluates the uncertainty by a single number, which conflicts with the fact that the deviation of the structural response changes as the design point moves. Therefore, the SF method has an extremely unstable accuracy. In contrast, the RBDO method considers the refined uncertainties, in which the deviation of the structural response keeps changing with the movement of the design point. Certainly, the uncertainty propagation method may not have a high calculation precision according to the structural response function. However, the RBDO method has done the best at describing the accurate uncertainty distribution or bounds of the structural response. Thus, the RBDO method is better than the SF method in terms of accuracy.

4.2.3 Sensitivity analysis

The sensitivity of the constraint condition for the two methods are different, because the two methods consider distinct factors. The SF method only considers the function between the middle value of the structural response and the design variables, while the RBDO method considers not only the mean value but also the deviation of the structural responses. The contribution of the uncertainty input to the uncertain structural response is also referred to as the uncertainty importance (UI) measure or global sensitivity. Many scholars have proposed computation methods for the global sensitivity index [37, 38]. Iman et al. [39] indicated that the UI index should have the characteristics of being unconditional, easy to understand, computable and stable. Based on these characteristics, he proposed three indexes and a computation method:

$$
\begin{aligned}
UI_1(i) &= \sqrt{\mathrm{Var}(Y) - \mathrm{E}\left[\mathrm{Var}(Y \mid X_i)\right]} \\
UI_2(i) &= \mathrm{Var}\left[\mathrm{E}(\ln Y \mid X_i)\right] / \mathrm{Var}(\ln Y) \\
UI_3(i) &= \left(Y_\alpha^* / Y_\alpha, Y_{1-\alpha}^* / Y_{1-\alpha}\right)
\end{aligned}
\tag{21}
$$

where $\mathrm{Var}(Y)$, Y_α and $Y_{1-\alpha}$ are the variance, α quantile and $1-\alpha$ quantile of the limit state function when the uncertainty inputs traverse all their domains, respectively; and $\mathrm{Var}(Y \mid X_i)$, Y_α^* and $Y_{1-\alpha}^*$ are the variance, α quantile and $1-\alpha$ quantile of the limit state function when the uncertainty inputs are defined as specific domains, respectively. From the above equations one can see that the global sensitivity is closely related to the variance of the response. However, the SF method is only related to the mean value or nominal value of the limit function response.

5. Numerical examples

In this section, the comparison of **the RBDO and SF methods** will be illustrated through the optimization design of a two-bar truss and a practical supersonic wing structure, in which the structural reliability of the design results of the PRBDO and NRBDO methods are analyzed based on the SF result.

5.1 Design of a two-bar truss

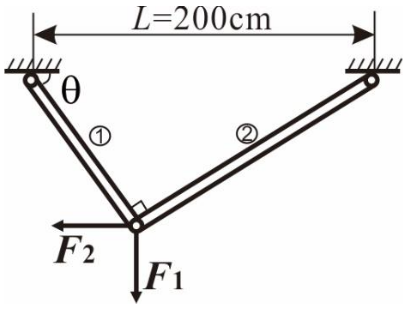


Fig. 8. A two-bar truss structural system.

A two-bar truss structure subjected to one uncertain vertical force and one uncertain horizontal force is shown in Fig. 8. The allowable stress of the structure and the external load are independently accounted for as uncertain parameters with normal distributions. The nominal values of the external forces are $F_1 = 10N$ and $F_2 = 5N$, while the reference coefficients of the variation about their nominal values are set to 10%. The mean and variance values of the allowable strength for the two bars are $\mu_{[\sigma_1]} = 6MPa,\ \mu_{[\sigma_2]} = 8MPa$ and $\sigma_{[\sigma_1]} = 0.6MPa,\ \sigma_{[\sigma_2]} = 0.8MPa$, respectively. The subscripts $[\sigma_1], [\sigma_2]$ represent the allowable values of the two bars. The two bars have the same density $\rho = 4.5g/cm^3$ with an angle $\theta = 60^o$. The cross-sectional areas of the truss are considered as design variables with respect to the initial values $A_i^0 = 3cm^2 (i = 1, 2)$. This paper adopts the series expansion and vertex methods to perform the uncertainty propagation analysis and reliability estimation. The optimization model is given by Eq. (22). The nonlinear programming by quadratic Lagrangian algorithm (NLPQL) [40] is utilized to perform the optimization. The objective function is to minimize the total mass of the truss system. The constraint of RBDO, which is the system reliability, is no less than that of the SF method, while the constraint of the SF is based on the SF of $n = 1.5$.

$$\begin{cases} \text{find} & \boldsymbol{A} = (A_1, A_2) \\ \text{min} & M = \sum_{i=1}^{2} \rho_i l_i A_i \\ \text{s.t.} & q_i = \dfrac{[\sigma]}{n} - S_i \geq 0\ ,\ when\ SF\ method \\ & \eta_1 \cdot \eta_2 \geq \eta_1^*\ ,\ when\ RBDO\ method \\ & A_i \geq 0.1cm^2,\ i = 1, 2 \end{cases} \tag{22}$$

The optimal cross-sectional areas of the SF, PRBDO and NRBDO models are separately shown in Table 2. The optimal scheme based on the PRBDO method is according to the optimization procedure shown in Fig. 3. Herein, the optimal results of the PRBDO method have the same probabilistic reliability index as the optimal results of the SF method. The optimal results of the NRBDO method have the same nonprobabilistic reliability index as the optimal results of the SF method. The iterations of the structural weight of the RBDO and SF methods are shown in Fig. 9, in which the iterations of the RBDO method start with the end of the iterations of the SF method. It can be seen that the SF method owns a deterministic design result with the total structural mass of $M = 2056.6g$. The solution of the PRBDO model is much less than that of the

SF method. Meanwhile, the optimal NRBDO solution is located between the two methods. From the point of the optimization results, the RBDO methods have a more notable weight reduction effect than the SF method.

Table 2 Optimal results of the structure with the three design methods.

	$A_1\ (cm^2)$	$A_2\ (cm^2)$	$\eta_1(\%)$	$\eta_2(\%)$	$\eta(\%)$	M (g)
Initial design	3.0000	3.0000	100	100	100	3688.3
SF	1.5401	1.7494	-	-	-	2056.6
PRBDO	1.6132	1.6252	99.61	99.41	99.02	1992.7
NRBDO	1.6962	1.6069	93.05	88.65	82.49	2015.7

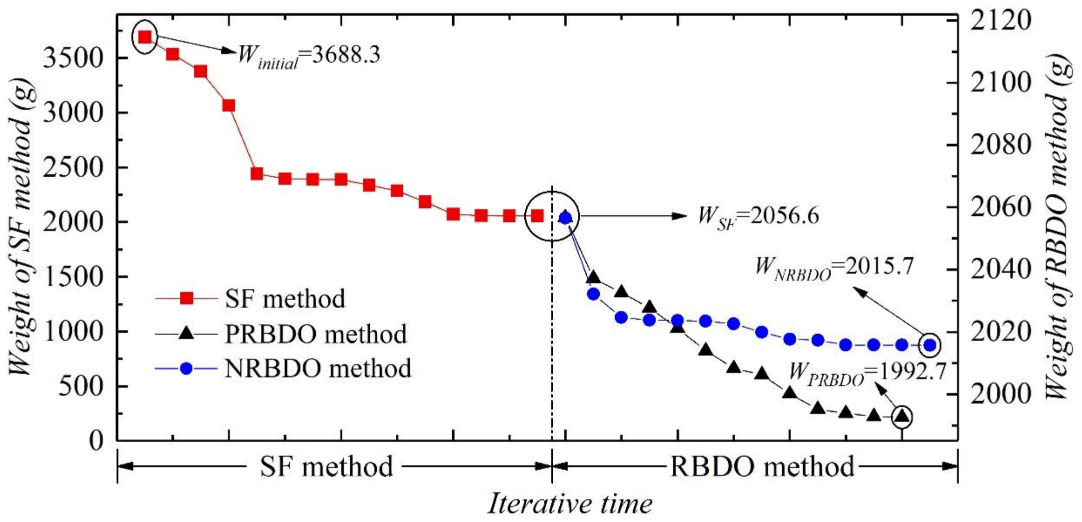


Fig. 9. Convergence curves of the two methods for the two-bar truss.

For the SF method used in this problem, the structural response is deterministic and can be represented as:

$$S_1 = (F_1 \cdot \cos(30^0) - F_2 \cdot \sin(30^0)) / A_1$$
$$S_2 = (F_1 \cdot \cos(60^0) + F_2 \cdot \sin(60^0)) / A_2 \quad (23)$$

The structural strength is expressed as $[\sigma]/n$. As a result, the design result can only be a deterministic value just as the green-line shown in Fig. 4, with the optimal design point being searched for from top to bottom until the critical conditions.

Correspondingly, the limit state functions of the two bars under the RBDO method are indicated as:

$$q_1 = [\sigma] - \frac{F_1 \cdot \cos(30^0) - F_2 \cdot \sin(30^0)}{A_1}$$
$$q_2 = [\sigma] - \frac{F_1 \cdot \cos(60^0) + F_2 \cdot \sin(60^0)}{A_2} \tag{24}$$

The structural allowable strength and the external load are variables with a distribution or interval, so the RBDO method can search for the optimal design point around the central value, as shown in curves ⑤, ② and ④.

In addition, it can be seen from Table 2 that the reliability of the two bars is 100% for the initial design. For the optimal scheme of the SF method, the reliability of the structure is 99.02% according to the process of Fig. 3. Meanwhile, the solution of the RBDO is better than the SF method with a lower target value. This is because properly decreasing the reliability of each bar (η_1, η_2) by the design of the areas of the two bars will lead to a higher reliability level (η) and a lower weight. Namely, the redistribution of reliability due to the comprehensive consideration of design variables between different bars is key to reducing the weight.

5.1.1 Time efficiency

The optimization flowchart of different methods is shown in Fig. 10, in which N expresses the number of deterministic structural analysis for the structural response g and m expresses the number of uncertainties. Herein, the uncertainties include the allowable strength $[\sigma_1]$, $[\sigma_2]$ and the external forces F_1, F_2 F_1, F_2; therefore m=4. From the figure, it can be seen that the RBDO methods need more than one deterministic analysis for g. In other words, the point of view of a single iteration, the SF method has an advantage over the RBDO method.

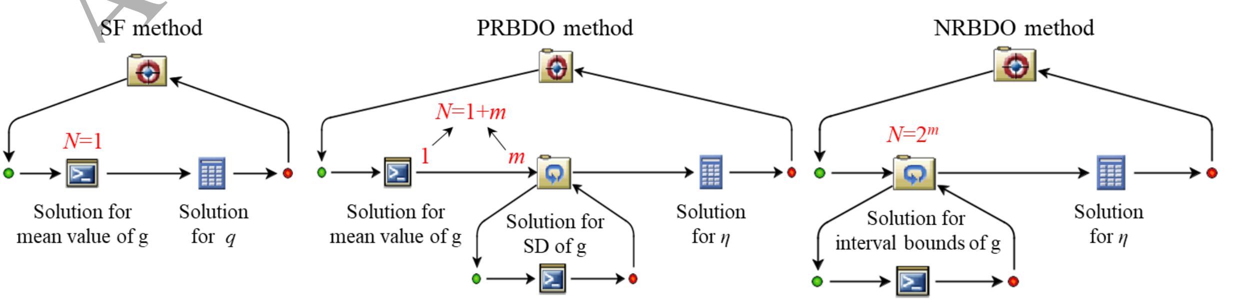


Fig. 10. Optimization flowcharts of the different methods.

5.1.2 Accuracy

To verify the accuracy of the reliability analysis, a Monte Carlo Simulation (MCS) is performed with 1e6 samples. For the probability reliability (PR) analysis, each sample is simulated by defining the allowable strength and external forces as random values. The PR of the MCS is defined by the number of samples that satisfies $h(\boldsymbol{P}^c, \Delta\boldsymbol{P}) - g(\boldsymbol{X}, \boldsymbol{P}^c, \Delta\boldsymbol{P}) \geq 0$ divided by 1e6. The PR of the first-order reliability method is calculated by Eqs. (9) and (10). For the nonprobability reliability (NR) analysis, each sample is simulated by defining an uncertain allowable strength and external forces as random values located in the interval bounds. The interval bounds of the stress are obtained as the minimum and maximum values of the stress samples. The calculated interval bounds of the stress are based on the vertex method. After the interval bounds of the stress are obtained, the NR are calculated via (13). The results of the MCS and the reliability analysis and the relative error are listed in Table 3. It can be seen that the errors are almost zero. Therefore, the accuracy of the reliability analysis method is proven.

Table 3 Comparison of the MCS and the reliability analysis

Type of reliability	$A_1\ (cm^2)$	$A_2\ (cm^2)$	η_1			η_2		
			MCS	**Calculated**	**Error**	**MCS**	**Calculated**	**Error**
PR	**1.5401**	**1.7494**	**0.9914**	**0.9915**	**0.01%**	**0.9987**	**0.9987**	**0.00%**
NR	**1.5401**	**1.7494**	**0.8756**	**0.8755**	**0.01%**	**0.9423**	**0.9421**	**0.02%**
PR	**1.6132**	**1.6252**	**0.9962**	**0.9961**	**0.01%**	**0.9942**	**0.9941**	**0.01%**
NR	**1.6962**	**1.6069**	**0.9306**	**0.9305**	**0.01%**	**0.8866**	**0.8865**	**0.01%**

After the accuracy is proven, the reliability analysis methods are considered to be the benchmark to test the constraint conditions. The reliabilities (η) and the limit state function (q) for the first numerical example are shown in Fig. 11 and Fig. 12. From Fig. 11, one can see that the design points of the SF method meet the reliability requirement. From Fig. 12, one can see that the design points of the RBDO methods cannot meet the constraint conditions since $q = \frac{h(\boldsymbol{P}^c)}{n_0} - g(\boldsymbol{X}, \boldsymbol{P}^c)$ is less than zero. It is known that the RBDO method is verified; in other words, the optimization schemes of the RBDO method satisfy the safety requirements and the constraint condition. However, these design schemes do not satisfy the constraint condition of the SF method. Thus, it can be considered that the SF method is inaccurate when measuring the structural safety. This is because the deviation is not constant in the optimization. Fig. 13 and Fig. 14 show the standard deviation (SD) and the radius in the RBDO process, respectively, which indicate that the deviation keeps changing as the design point changes.

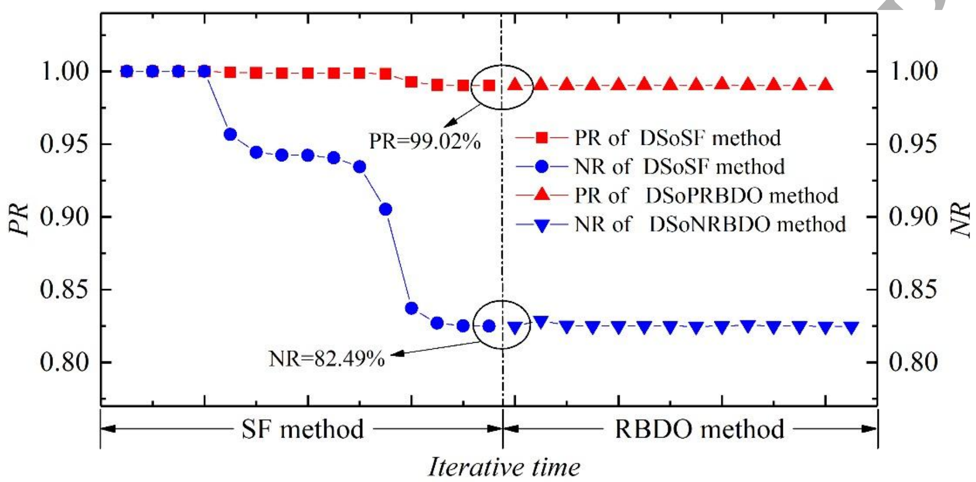


Fig. 11. Reliability (η) with respect to the iterative time for the two methods

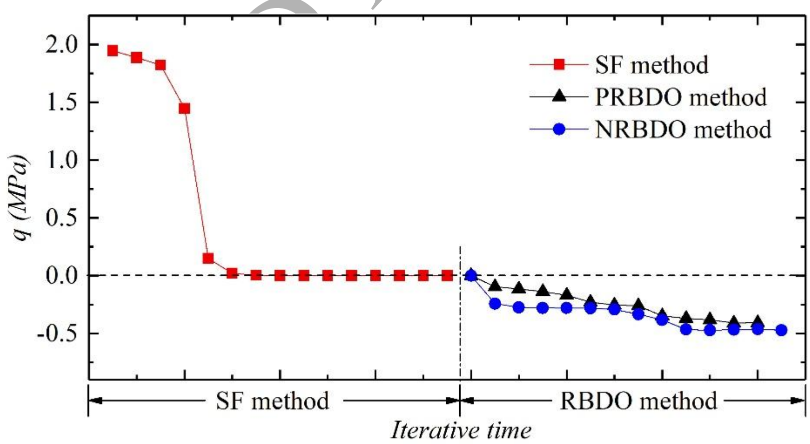
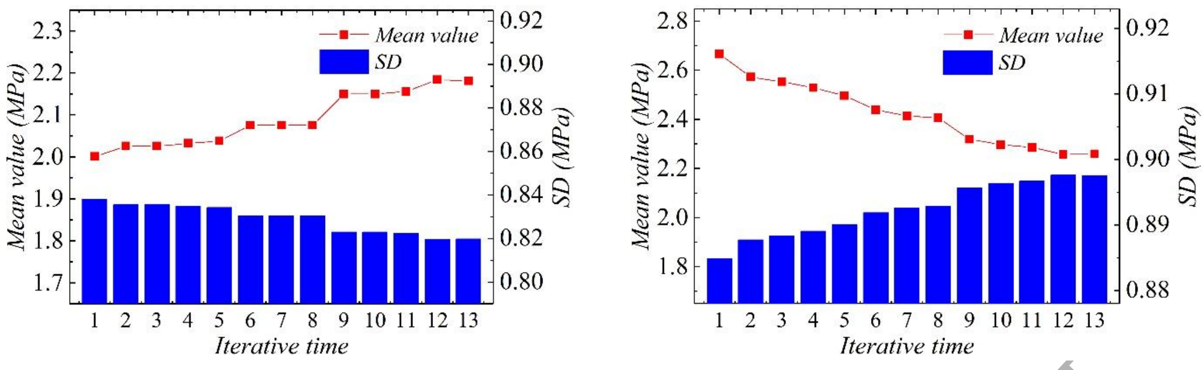
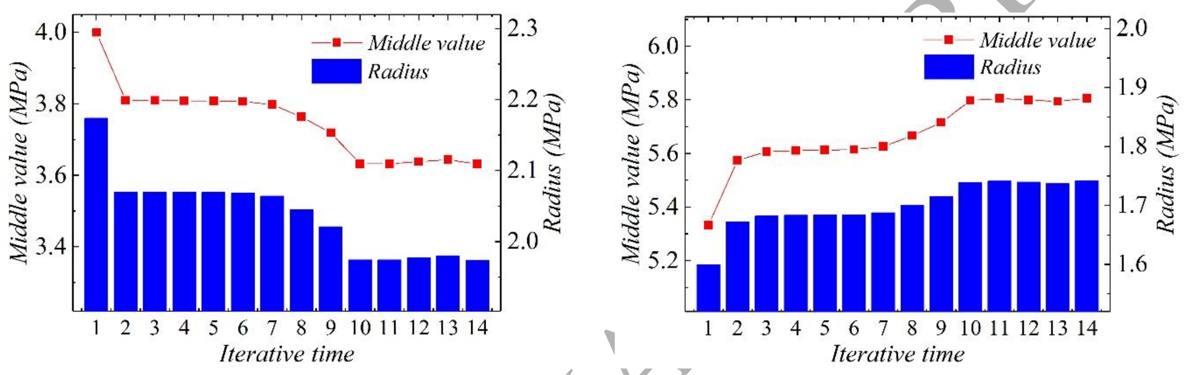


Fig. 12. Limit state function (q) with respect to the iterative time for the two methods



(a) (b)

Fig. 13. Mean value and SD of stress: (a) bar 1, (a) bar 2.

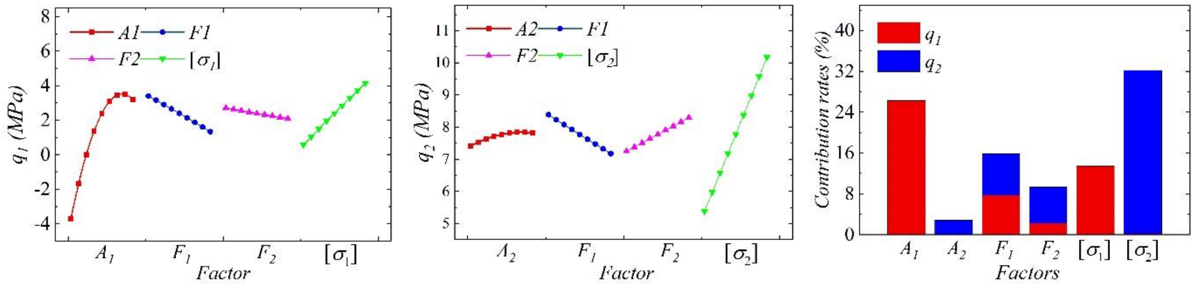


(a) (b)

Fig. 14. Middle value and radius of stress: (a) bar 1, (a) bar 2.

5.1.3 Sensitivity

In the sensitivity analysis, the sensitivity of the design variables and the uncertainty variables to the limit state function (q_1, q_2) are analyzed. The effect trends and Pareto graph are shown in Fig. 15. It can be seen that the design variables and the uncertainties simultaneously affect the q_1 *and* q_2 . From Fig. 15 (a) and (b) one can see that the limit state function has peak values in the domain of the design variables. The limit state function has a linear and monotonous relationship with the uncertainties. Thus the series expansion and vertex methods can be utilized to calculate the interval bounds. From Fig. 15 (c) , one can see that the design variable A_1 has a greater contribution to q_1 than to q_2 , and the uncertainties have different contribution rates to the limit state function. Table 4 shows the uncertainty importance (UI) of the different uncertainties based on Eq. (21). It can be seen that the uncertainties of the allowable strength have the most important contribution rates to the limit state function, which is due to the coefficient of the uncertain variables. Meanwhile, one can see that different uncertain variables have different sensitivities. The two kinds of UI indexes have the same tendency in reflecting the importance degree. Thus, the different combinations will emerge at different design points, and the deviations may be reduced.



(a) (b) (c)

Fig. 15. Sensitivity analysis for the design variables and uncertainties: (a) Effect trends of q_1; (b) Effect trends of q_2; (c) Pareto graph for the structural response.

Table 4 Global sensitivity of the uncertainties

	q_1			q_2		
	$[\sigma_1]$	F_1	F_2	$[\sigma_2]$	F_1	F_2
UI_1	0.600	0.289	0.083	0.800	0.167	0.144
UI_3	(1.211,0.879)	(1.033,0.981)	(1.003,0.999)	(1.267,0.843)	(1.006,0.990)	(1.004,0.997)

5.2 Design of a supersonic wing structure

For the sake of demonstrating the feasibility of the developed method in practical engineering structures, a supersonic wing structure is designed in this section. The simplified structural model consists of a leading edge, trailing edge, wing beam, wing rib and skins, which are presented in Fig. 16. The aerodynamic lifting force is represented by a uniform pressure, which is applied to the surface of the lower skin. The wing structure is made of an aluminum alloy with a Young's modulus E=70GPa, a structural strength $\sigma_s = 462MPa$, Poisson's ratio μ=0.33, and a mass density ρ=2.8g/cm^3. In this example, the allowable strength, the Young's modulus and the Poisson's ratio are defined as the random variables. The random parameters are considered to obey normal distributions, and the reference coefficients of variation about their nominal values are set to 10%. The SF of the structural design is defined as $n = 1.5$.

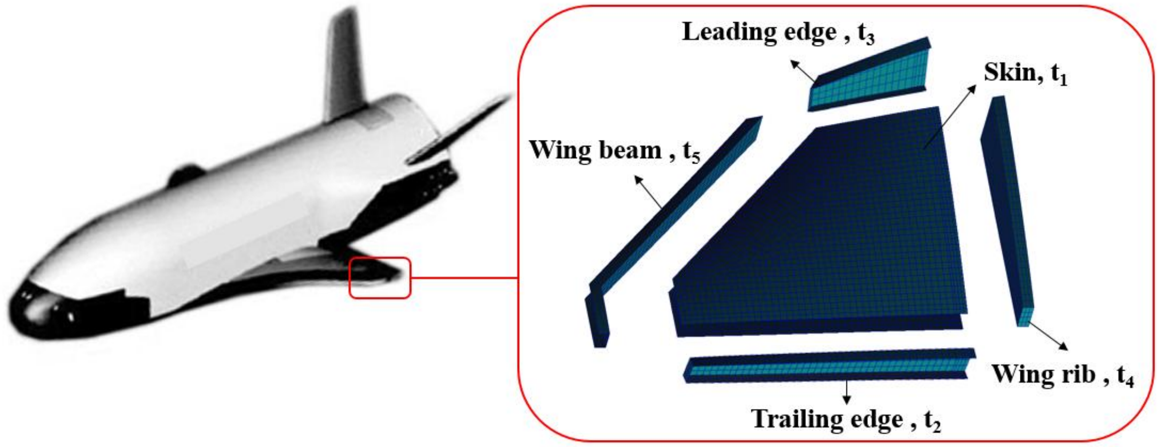


Fig. 16. Overall optimization model of wing structure.

The thicknesses of the wing components are design variables and are shown in Fig. 16. For the optimization model of the SF method, the constraint conditions are the maximum structure stress and the displacement lower than the allowable values to guarantee the safety of the wing structure, where the optimization objective is to minimize the structural weight. The wing structure optimization model of the SF method is shown below.

$$\begin{cases} \min & W \\ \text{find} & \boldsymbol{t}=[t_1,t_2,t_3,t_4,t_5] \\ \text{s.t.} & dis \le [dis]_{\text{cr}} \\ & \sigma \le [\sigma] \end{cases} \tag{25}$$

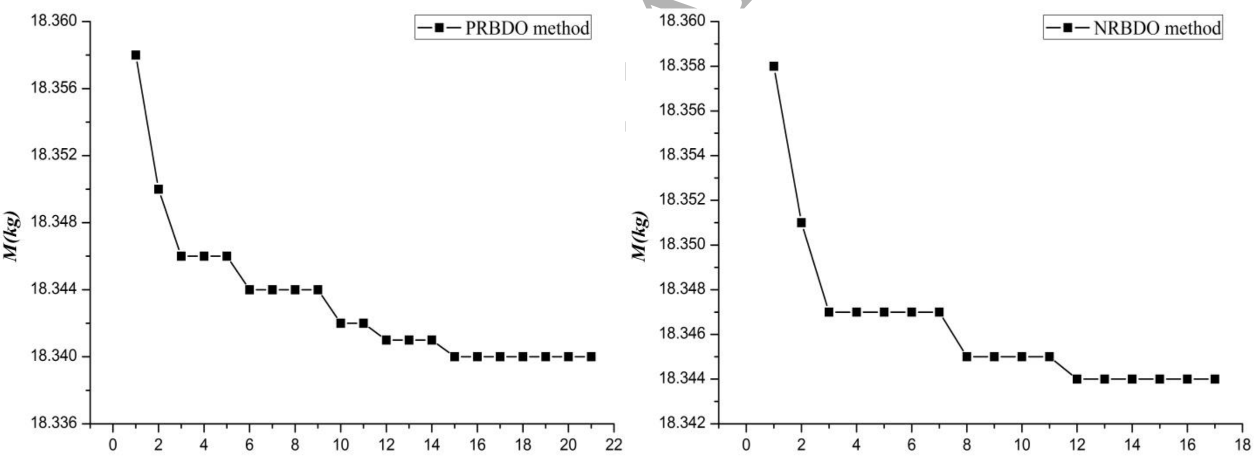
In the model above, W is the structural weight and $\boldsymbol{t}=[t_1,t_2,t_3,t_4,t_5]$ is the vector matrix of the design variables. $[dis]_{\text{cr}}$ and dis are the allowable and response values of the displacement, respectively. $[\sigma]$ represents the allowable structure stress and σ indicates the stress when the wing structure is loaded.

In the optimization model of the RBDO method, the constraints are that the reliability of the structural response is not less than the reliability of the optimal design result by the SF method (η^*_{dis} and η^*_{σ}). η_{dis} and η_{σ} are the structural reliability of the displacement and stress, respectively. The difference between the NRBDO and the PRBDO methods is the calculation method of the reliability index, which uses two representative calculation methods in the reliability theory. In other respects, the optimization objective and the design variables are the same as the deterministic model, and the model of the RBDO method is shown as follows

$$\begin{cases} \min & W \\ \text{find} & \boldsymbol{t} = [t_1, t_2, t_3, t_4, t_5] \\ \text{s.t.} & \eta_{dis} \ge \eta_{dis}^* \\ & \eta_\sigma \ge \eta_\sigma^* \end{cases} \tag{26}$$

On the basis of the calculation process shown in Fig. 3, the optimization of the wing structure with three methods can be solved through the same engineering algorithm, which will be more conducive to comparing the three design methods. The series expansion method is adopted to perform the uncertainty propagation analysis and reliability estimation, and the NLPQL algorithm is adopted to solve the optimization model. The convergence curves of the RBDO methods are presented in (a) (b)

Fig. 17 and the optimal results are listed in Table 5. Herein, the optimal results of the PRBDO method have the same probabilistic reliability index as the optimal results of the SF method, and the optimal results of the NRBDO method have the same nonprobabilistic reliability index as the optimal results of the SF method.



(a) (b)

Fig. 17. Convergence curves of RBDO methods: (a)PRBDO method; (b)NRBDO method.

Table 5 The Optimal results of wing structure with three design methods.

	t_1 (mm)	t_2 (mm)	t_3 (mm)	t_4 (mm)	t_5 (mm)	η_{dis}	η_σ	M (kg)
SF	3.722	2.614	11.983	6.948	1.576	-	-	18.358
PRBDO	3.722	2.722	11.871	6.815	1.654	0.962	0.985	18.340
NRBDO	3.739	2.631	11.894	6.858	1.572	0.946	0.974	18.344

As shown in the table above, the results of the two RBDO methods are both better than that of the SF method with the weight reduction effect of the wing structure. In addition, the PRBDO method can provide more refined optimization results compared to the NRBDO method, but it requires more distribution information of the design parameters. The effect of the structural weight reduction is limited because of the strict constraints, which means the reliability constraint conditions of the RBDO method are calculated from the optimal design result of the SF method.

The reasons why the RBDO method can determine a better solution is that there is an inconsistency in the reliability of the structural parts in spite of the same SF, which means that the structural components have different structural safety levels. By comparison, the RBDO method can coordinate the reliability distribution between the components and expand the design space, which means it is more likely to find a better solution, with the conditions when the structure has a better solution discussed in Sec 4.4. Again, this example confirms that the RBDO method has a better weight reduction effect than the SF method.

6. Conclusions

The **RBDO and SF methods** are compared in this paper from the aspects of the optimization results and optimization process. Three conclusions can be obtained through the discussion and engineering examples:

1) Normally, the result of the SF method is a conservative local solution because it ignores the variance of the design variables; in contrast, the RBDO method can obtain the global optimal solution in the structural design due to the comprehensive consideration of the mean value and variance factor;
2) The structures designed by virtue of the RBDO method usually present reasonably when adjusting the reliability level of every structural component during the design process to further reduce the structural weight;
3) The SF method has deficiencies in describing the structural uncertainties in the design process, because it expresses all uncertainties with a constant coefficient, which is demonstrated as inaccurate in the numerical example. In contrast, the RBDO method can describe the refined uncertainties, which is closer to reality.

The RBDO method has advantages in handling the uncertainties in the structural design.

Meanwhile, the proposed optimization flowchart in this paper promotes the application of the RBDO methods. Although the RBDO method is more reasonable for the structural design and has undergone decades of development, the variable transmission and coupling in engineering optimization still requires further study, especially when multiple disciplines are involved.

Acknowledgments

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References

[1] G.Y. Wang. On the development of uncertain structural mechanics. Adv. Appl. Mech. 32 (2) (2002) 205-211.
[2] I. Elishakoff. Safety factors and reliability: friends or foes? Springer Science & Business Media. Berlin. 2012.
[3] B. Ellingwood. T.V. Galambos. Probability-based criteria for structural design. Struct. Saf. 1 (1) (1982) 15-26.
[4] I. Elishakoff. B. Ferracuti. Fuzzy sets based interpretation of the safety factor. Fuzzy. Set. Syst. 157 (18) (2006) 2495-2512.
[5] Y. Benhaim. A non-probabilistic concept of reliability. Struct. Saf. 14 (4) (1994) 227-245.
[6] X.J. Wang. L. Wang. Z.P. Qiu. Safety estimation of structural systems via interval analysis. Chinese. J. Aeronaut. 26 (3) (2013) 614-623.
[7] V. Verderaime. Aerostructural safety factor criteria using deterministic reliability. J. Spacecr. Technol. 30 (2) (1993) 244-247.
[8] C.A. Cornell. Safety specification based on second-moment reliability analysis, in: International Symposium on Concepts of Safety of Structures and Methods of Analysis, London, 1969, pp.235-245
[9] A.M. Hasofer. N.C. Lind. Exact and invariant second-moment code format. J. Eng. Mech. 100 (1) (1974) 111-121.
[10] R. Rackwitz. B. Flessler. Structural reliability under combined random load sequences. Comput. Struct. 9 (5) (1978) 489-494.
[11] K. Breitung. Asymptotic Approximations for Multinormal Integrals. J. Eng. Mech. 110 (3) (1984) 357-366.
[12] J.W. Lincoln. Risk assessment of an aging military aircraft. J. Aircraft. 22 (8) (2012) 687-691.
[13] I. Elishakoff. F. Elettro. Interval, ellipsoidal, and super-ellipsoidal calculi for experimental and

theoretical treatment of uncertainty: Which one ought to be preferred? Int. J. Solids. Struct. 51 (7-8) (2014) 1576-1586.
[14]X. Chen. X.J. Wang. L. Wang. Q.H. Shi. Y.L. Li. Uncertainty quantification of multi-dimensional parameters for composite laminates based on grey mathematical theory. Appl. Math.Model. 55 (2018) 299-313.
[15]C. Jiang. R.G. Bi. G.Y. Lu. X. Han. Structural reliability analysis using non-probabilistic convex model. Comput. Method. Appl. M. 254 (2) (2013) 83-98.
[16]L. Wang. X.J. Wang. X. Chen. R.X. Wang. Time-Dependent Reliability Modeling and Analysis Method for Mechanics Based on Convex Process. Math. Probl. Eng. 2015 (1) (2015) 1-16.
[17]Z. Meng. H.L. Zhou. G. Li. D.X. Yang. A decoupled approach for non-probabilistic reliability-based design optimization. Comput. Struct. 175 (2016) 65-73.
[18]Y. Ben-Haim. I. Elishakoff. Discussion on: A non-probabilistic concept of reliability. Struct. Saf. 17 (3) (1995) 195-199.
[19]Z.P. Qiu. I. Elishakoff. Antioptimization of structures with large uncertain-but-non-random parameters via interval analysis. Comput. Method. Appl. M. 152 (3) (1998) 361-372.
[20]L. Wang. Y.R. Cai. D.L. Liu. Multiscale reliability-based topology optimization methodology for truss-like microstructures with unknown-but-bounded uncertainties. Comput. Method. Appl. M. 339 (2018) 358-388.
[21]Z.P. Qiu. X.J. Wang. Comparison of dynamic response of structures with uncertain-but-bounded parameters using non-probabilistic interval analysis method and probabilistic approach. Int. J. Solids. Struct. 40 (20) (2003) 5423-5439.
[22]Z.P. Qiu. P.C. Mueller. A. Frommer. The new nonprobabilistic criterion of failure for dynamical systems based on convex models. Math. Comp. Model. 40 (1) (2004) 201-215.
[23]J.Y. Ching. Equivalence between reliability and factor of safety. Probabilist Eng Mech. 24 (2) (2009) 159-171.
[24]J.W. Bristow. P.E. Irving. Safety factors in civil aircraft design requirements. Eng. Fail. Anal. 14 (3) (2007) 459-470.
[25]A. Sarja. E. Vesikari. Durability design of concrete structures. CRC Press. Boca Raton. 1994.
[26]X.Y. Qu. R. Haftka. Reliability-based Design Optimization Using Probabilistic Safety Factor, in: 47th AIAA/ASMW/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Chicago, 2006,
[27]G.E. Muller. C.J. Schmid, Factor of Safety - USAF Design Practice, 1978.
[28]W. Heath. The changing scene of structural airworthiness. The Aeronautical Journal. 84 (831) (1980) 81-92.
[29]L. Wang. D.L. Liu. Y.W. Yang. X.J. Wang. Z.P. Qiu. A novel method of non-probabilistic reliability-based topology optimization corresponding to continuum structures with unknown but bounded uncertainties. Comput. Method. Appl. M. 326 (2017) 573-595.
[30]Q. Li. Z.P. Qiu. X.D. Zhang. Eigenvalue analysis of structures with interval parameters using the second-order Taylor series expansion and the DCA for QB. Acta. Mech. Sinica. 31 (6) (2015) 845-854.
[31]J. Li. J.B. Chen. Stochastic dynamics of structures. Wiley. New Jersey. 2010.
[32]M.H. Xu. J.K. Du. C. Wang. Y.L. Li. A dimension-wise analysis method for the structural-acoustic system with interval parameters. J. Sound. Vib. 394 (2017) 418-433.

[33] X.J. Wang. Z. Qiu. I. Elishakoff. Non-probabilistic set-theoretic model for structural safety measure. Acta. Mech. 198 (1-2) (2008) 51-64.
[34] T.H. Lee. K.M. Mosalam. Seismic demand sensitivity of reinforced concrete shear‐wall building using FOSM method. Earthq. Eng. Struct. D. 34 (14) (2005) 1719-1736.
[35] J. Li. J.B. Chen. Probability density evolution method for dynamic response analysis of structures with uncertain parameters. Comput. Mech. 34 (5) (2004) 400-409.
[36] Q. Shi. X. Wang. R. Wang. X. Chen. Y. Ma. An interval updating model for composite structures optimization. Compos. Struct. 209 (2019) 177-191.
[37] J.C. Helton. J.D. Johnson. C.J. Sallaberry. C.B. Storlie. Survey of sampling-based methods for uncertainty and sensitivity analysis. Reliab. Eng. Syst. Safe. 91 (10–11) (2006) 1175-1209.
[38] C.B. Storlie. L.P. Swiler. J.C. Helton. C.J. Sallaberry. Implementation and evaluation of nonparametric regression procedures for sensitivity analysis of computationally demanding models. Reliab. Eng. Syst. Safe. 94 (11) (2009) 1735-1763.
[39] R.L. Iman. S.C. Hora. A Robust Measure of Uncertainty Importance for Use in Fault Tree System Analysis. Risk. Anal. 10 (3) (2010) 401-406.
[40] N. Hu. J.G. Yang. P.L. Zhou. Optimisation of injection-related parameters using the non-linear programming by quadratic Lagrangian algorithm for a marine medium-speed diesel engine. Int. J. Engine. Res. 19 (5) (2018) 570-583.