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Cyclic plastic zone-based notch analysis and damage evolution model for fatigue life prediction of metals

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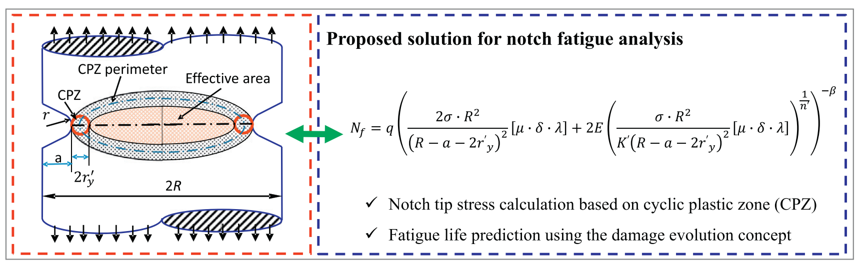
HIGHLIGHTS

- New closed form notch tip stress calculation method based on cyclic plastic zone
- Proposed damage evolution model for notch fatigue life prediction
- Notch tip stress relates with both the notch geometry and material property.
- Proposed model yields better predictions of three materials than other two models.

GRAPHICAL ABSTRACT



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ABSTRACT

Fatigue strength analysis of critical components plays a vital role for ensuring structural integrity and operational reliability of major equipment. In notched components, the concept of cyclic plastic zone (CPZ) is commonly utilized for fatigue cracking analysis, in which the CPZ size normally relates to the material strength. In particular, materials with higher yield stress have shown smaller CPZ size and vice versa. According to this, a new approach for determining closed-form stress at the notch tip is proposed by considering the size of cyclic plastic zone, which can be used for the notch tip stress evaluation along the load direction under cyclic loadings. By implementing FE analysis, experimental data of 304 stainless steel, 40Cr steel and Ti-6Al-4V alloy are utilized for model validation and comparison. Results show that the scope of damaged CPZ alters the notch tip stress under fatigue loadings, and the proposed model yields better correlation of fatigue life predictions with experimental results of the three materials than other two models.

1. Introduction

For engineering structures/components subjected to cyclic loadings, their structural strengths ordinarily degrade with the process of fatigue damage accumulation. In general, fatigue damage is the main source of subsequent failures of these components [1–6], in which the process of fatigue failure includes crack initiation and propagation till eventual fracture [7–11]. Fatigue crack propagation life is usually estimated by the following solutions, including: i) models relate the stress intensity range with crack growth rate like the Paris-Erdogan model [12–14], ii) models monitor crack tip opening displacement in which crack propagation occurs when the crack tip is fully opened [15] and iii) models relate advancement of crack tip deformation in the damage evolution model [7,16–19].

Until now, various fatigue life prediction methods have been developed, including methods based on continuum damage mechanics (CDM) [2,4,12,20], fracture mechanics using the theory of critical

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Nomenclature

a	notch depth
b	fatigue strength exponent
c	fatigue ductility exponent
D	damage variable
E	elastic modulus
E_r	error distribution
F	geometric factor
g	damaged notch depth to damaged notch radius factor
K	circulation strengthening coefficient
K'	cyclic strength coefficient
σ_m	mean stress
K_t	stress concentration factor
ΔK	stress intensity factor range
M_o	material constant
N	fatigue life
N_f	fatigue fracture life
N_f^{es}	estimated fatigue life
N_f^{ex}	experimental fatigue life
n	stain hardening index
n'	cyclic stain hardening coefficient
P	loading condition parameter
p	damage notch depth to nominal radius ratio
q	loading condition parameter
r	notch radius
r_y'	cyclic plastic zone size
$\Delta\varepsilon$	total strain range
$\Delta\varepsilon_e$	elastic stain range
$\Delta\varepsilon_p$	plastic stain range
ε_y	yield strain
ε_f'	fatigue ductility coefficient
σ	applied stress
σ_{max}	maximum stress
σ_{min}	minimum stress
σ_a	stress amplitude
σ_n	nominal stress
σ_y	yield stress
σ_f'	fatigue strength coefficient
α	material constant
δ	crack (notch) extension coefficient
λ	undamaged to damaged area radius ratio
γ	material constant
R	specimens' outer radius
FEA	finite element analysis
LSM	least square method
MPZ	monotonic plastic zone
CPZ	cyclic plastic zone

distance (TCD) [3,21], stress gradient [3,22,23] and strain energy [24–26]. Among them, Manson-Coffin equation has been extensively used for fatigue life prediction under uniaxial loading conditions [27–29]. Liu et al. [2] introduced a model for life prediction of the whole cracking process. By coupling the TCD and stress gradient concepts, Andrea et al. [3] developed an approach for fatigue life prediction of notched components. Based on non-linear CDM theory, Dattoma et al. [1] elaborated a model for fatigue life prediction under variable amplitude loadings which considers the effect of load sequence. Aleksander et al. [30] presented an algorithm to describe the variability of coefficients which vary with the applied cycles when applying the critical plane-based approaches for multiaxial fatigue life prediction. Recently, Aid et al. [4] conducted fatigue damage accumulation modeling and life prediction under variable amplitude loadings.

The reason for crack initiation at the notch region is the continuous buildup of cyclic plastic zone (CPZ) [31]. In general, fatigue crack growth is integrally controlled by the plastic zones which exists both in the vicinity of a propagating crack and around (tip) of the adjoining surfaces [7,9,31–36]. Numerous studies have shown that failure points of structures generally locates at their geometric discontinuities, which present severe local plastic deformations after cyclic loadings [37–45]. According to this, various studies on CPZ-based fatigue analysis have been conducted. Among them, Shi et al. [39] established a theoretical model for low cycle fatigue (LCF) analysis based on plastic strain energy ahead of the crack tip and the effective CPZ. By conducting FE simulation under plane strain condition, Surajit [40] came up with a numerical model to describe the effect of inclusions presence on size and shape of monotonic plastic zone (MPZ) and CPZ at the crack tip. In the study of deformation in the crack tip plastic zone and its role in crack propagation, Gao et al. [41] found that heterogeneous slip and secondary microcracks are the main features of the fatigue crack tip plastic zone. Vallellano et al. [42] provided a micromechanical description of small crack propagation based on the successive blocking of the MPZ and CPZ at the microstructural barriers.

In addition, various researches have been carried out to establish a link between fatigue life and notch tip stress by finite element analysis (FEA) [46–49], more recent advances on notch analysis in metal fatigue can be found in [50]. Gates and Fatemi [22] pointed out that accurate modeling of the material stress-strain response at critical locations is the main factor for accurate fatigue life predictions, in which the notch tip stress is determined based on known nominal loads, notch geometries and load histories. However, it is worth noting that CPZ alters the notch geometry and dimensions under fatigue loadings [31,32,51]. Besides, notch tip stress also varies with the notch geometry. Therefore, the influence of CPZ on the notch tip stress and fatigue life deserves an intensive study for fatigue analysis of engineering components.

This study attempts to present a damage mechanics model with new closed-form notch tip stress for fatigue life prediction of notched components. In particular, a novel approach on determining the notch tip stress is elaborated by considering the effect of CPZ size on the crack extension factor and notch geometry. The damage evolution process is modeled under uniaxial loadings together with the proposed notch tip stress and Massing's hypothesis. Experimental results of 304 stainless steel, 40Cr steel, Ti-6Al-4V alloy are utilized for model validation and comparison. The rest of this work is structured as follows: Section 2 gives a brief review on the CPZ, damage evolution, Manson-Coffin equation and Liu's model; Section 3 elaborates the derivation of the new model based on CPZ size and damage evolution; Section 4 conducts model validation by experimental results of notched specimens; Finally, Section 5 draws the conclusions.

2. Theoretical background

2.1. Cyclic plastic zone

As aforementioned, various studies have been conducted to explain how notched parts fail due to CPZ which forms accumulation of damaged blocks during cyclic loadings [16,31,32,52–54]. When a material block is fully damaged, then final failure of the material block and fatigue crack propagates see Fig. 1. Therefore, it is essential to understand the mechanism of cyclic plastic deformation at the crack tip and choose or develop a proper damage accumulation model for fatigue crack growth analysis.

Fig. 2 depicts the plastic zone concept at the tip of an advancing crack, in which the plastic zone size is developed when the load reached the maximum tension and compression at the points D and E per loading cycle (see Fig. 2(a)). Meanwhile, Fig. 2(b) shows the notch tip stress distribution in the monotonic plastic zone (MPZ), when the axial load reduces to its minimum (the compression load point E), the local stress

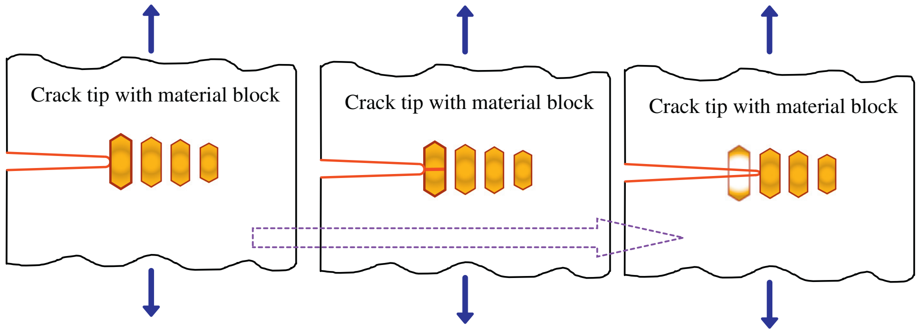

Fig. 1. Schematic diagram of damage accumulation and fatigue crack growth.

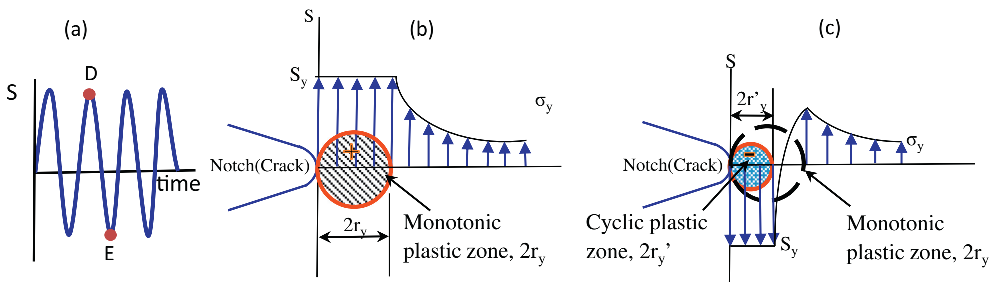

Fig. 2. Schematic of the plastic zone at the tip of an advancing crack. (a) loading cycle, (b) monotonic plastic zone, (c) cyclic plastic zone.

reduces to the value less than MPZ. This cyclic loading process causes the development of CPZ [7–9], see Fig. 2(c).

In general, fatigue crack growth is integrally controlled by the plastic zone. CPZ has shown a direct and vital role in controlling both fracture and crack growth [8,9]. Wang [31] and Liu et al. [55] put forward that the CPZ facilitates fast growing of cracks and controls mechanical driving force for crack propagation. The size of the CPZ depends on the strength of the material. Specifically, materials with high yield stress normally have small CPZ size and vice versa [8,9,51].

Irwin [51] found that the formation of a CPZ affects the crack geometry when it is longer than its physical size, and then he estimated the size of the CPZ by:

$$r'_y = \frac{1}{8\pi}\left(\frac{\Delta K}{\sigma_y}\right)^2 \text{ for plane stress condition} \quad (1)$$

$$r'_y = \frac{1}{24\pi}\left(\frac{\Delta K}{\sigma_y}\right)^2 \text{ for plane strain condition} \quad (2)$$

Note that the CPZ size depends on the applied stress, yield stress of the material and notch geometries [56–59]. Until now, various empirical formulas on the size of CPZ [56–62] have been developed, some of them are summarized in Table 1. Note that these empirical formulas yield different CPZ sizes. Among them, both Bathias et al. [60] and Pineau et al. [57] formulations predict oversized CPZ, while Park's et al. [59], Chapatti et al. [61] and Edmunds et al. [62] ones for smaller size of CPZ.

In Fig. 3(a), the effective area is depicted as the cross sectional area at the notch point. Fig. 3 plots the damaged effective area and extended notch radius by CPZ, in which a 3D schematic diagram of cyclic plastically damaged cross section is depicted in Fig. 3(a) by subtracting the plastically damaged area from the notch cross section and a 2D schematic diagram in Fig. 3(b). When the plasticity alters the geometry around the perimeter of the notch tip, the effective area reduces in relation to the size of CPZ under cyclic loadings. In earlier works, Irwin [51] pointed out that CPZ changes the notch geometry and size. However, Gates and Fatemi [22] found that notch tip stress greatly depends on the notch geometry and its dimensions. Thus, the size of CPZ affects the notch tip stress. Accordingly, this study attempts to investigate the effect of CPZ on the notch tip stress and fatigue life prediction.

2.2. Damage evolution model

During fatigue life evaluation, it is important to link the formulation between fatigue damage accumulation and the material properties [1,12]. Lemaitre [20] and Liu et al. [2] correlated fatigue damage with the strain of damaged material and applied loads. Chaboche [63] developed a damage evolution expression which associates the fatigue life with the total stress amplitude and material constants, and a constitutive formula showing the two damage relations are presented as follows:

$$\sigma = E(1-D)\varepsilon \quad (3)$$

Table 1
Empirical formulas for the cyclic plastic zone size.

Reference	Cyclic plastic zone size
Nicholls and Martin [56]	$r'_y = a\left(\frac{\sigma_{app}^2}{\sigma_y^2 - \sigma_{app}^2}\right)$
Bathias and Pelloux [60]	$r'_y = 0.1\left(\frac{\Delta K}{\sigma_y}\right)^2$
Pineau and Pelloux [57]	$r'_y = 0.053\left(\frac{\Delta K}{\sigma_y}\right)^2$
Saxena and Antolovich [58]	$r'_y = \alpha\left(\frac{\Delta K}{\sigma_y}\right)^{2+s}$
Park et al. [59]	$r'_y = \frac{\pi}{144}\left(\frac{\Delta K}{\sigma_y}\right)^2$
Chapetti et al. [61]	$r'_y = \frac{1}{12\pi}\left(\frac{\Delta K}{\sigma_y}\right)^2$
Edmunds and Willis [62]	$r'_y = \frac{1}{24\pi}\left(\frac{\Delta K}{\sigma_y}\right)^2$

σ_{app} represents the applied stress, α and s denotes correlation and variation coefficients, respectively.

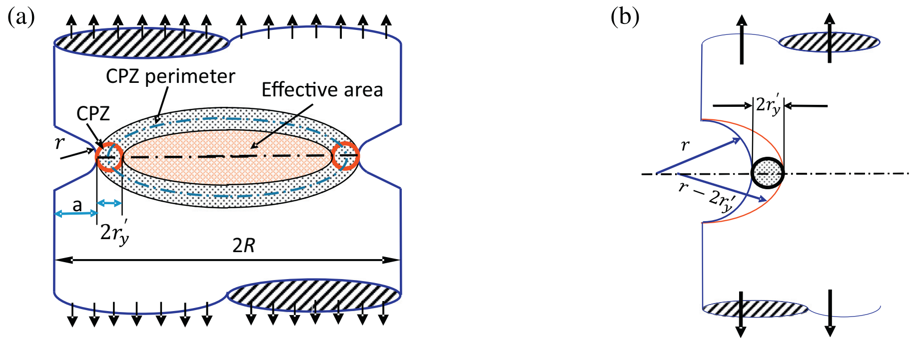

Fig. 3. Schematic diagram CPZ around the perimeter of notched specimen and cross sectional effective area (a) 3D view and (b) 2D view of the Notch radius (extended due to CPZ).

where E is the elastic modulus of material of interest, ε is the strain of damaged material and D donates the cumulative damage:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha(\sigma_{max},\sigma_{min})}\left[\frac{\sigma_a}{M_o}(1-b\sigma_m)(1-D)\right]^{\beta}\delta \mathrm{N} \tag{4}$$

where $\sigma_a = \frac{1}{2}(\sigma_{max}-\sigma_{min})$ is the stress amplitude, b, β and M_o are material constants, σ_{max} and σ_m are respectively the maximum and mean stress, and the exponent α depends on the loading parameters (σ_{max},σ_m).

Under fully reversible loadings ($R=-1$), the following relation holds with $\sigma_a=\sigma_{max}$:

$$dD = (1-D)^{-\beta}\left[1-(1-D)^{\beta+1}\right]^{\alpha}\left(\frac{\sigma_t}{M_o}\right)^{\beta}\mathrm{dN} \tag{5}$$

2.3. Manson-Coffin equation

Manson-Coffin equation has been widely used in fatigue life prediction under constant amplitude loadings, which establishes an intact relationship between the total strain range and fatigue life [2], [27–29]. In particular, as presented in Eqs. (6)–(8), the total strain range of the Manson-Coffin equation is the summation of elastic and plastic parts:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} \tag{6}$$

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma_f'}{E}(2N_f)^b \tag{7}$$

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f'(2N_f)^c \tag{8}$$

The plastic and elastic strain range can be obtained from the elasto-plastic stress-strain response at the critical location (like the notch tip).

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \tag{9}$$

where $\Delta\varepsilon$ is the total strain range, $\Delta\varepsilon_e$ is the elastic strain range, $\Delta\varepsilon_p$ is the plastic strain range, N_f is the fatigue life, σ_f' and b are respectively fatigue strength coefficient and exponent, ε_f' and c are the fatigue ductility coefficient and exponent, respectively.

2.4. Liu's model

Liu et al. [2] predicted the whole fatigue life of notch components based on CDM, in which fatigue lives of notched specimens with different stress concentration factors can be estimated by (just the Liu model for short):

$$N_f = P\left[\frac{\sigma_n(2K_t+3)(R-a)^2}{5ER^2}\right]^{-\gamma} \tag{10}$$

where P and γ are respectively parameters related to loading condition and material constant; R and a are the outer radius and notch depth of the specimen, respectively.

3. Proposed model

In this section, a new approach for closed-form stress at the notch tip is put forward by considering the size of CPZ and comparing with FEA results during notch tip stress calculations. Specially, Fig. 4(a) plots FE

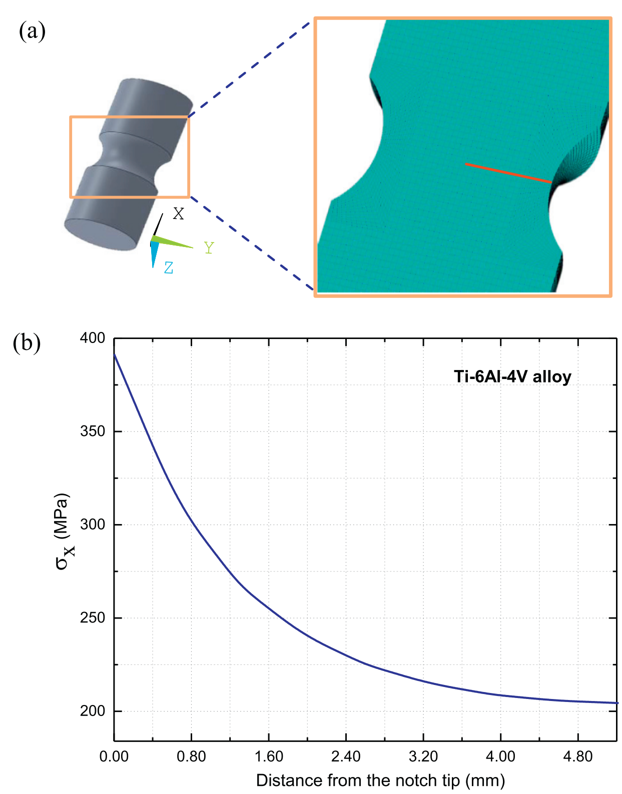

Fig. 4. (a) FE mesh, (b) $\sigma_X$component stress distribution along the radial direction on the minimum cross section of notched specimen under cyclic loadings.

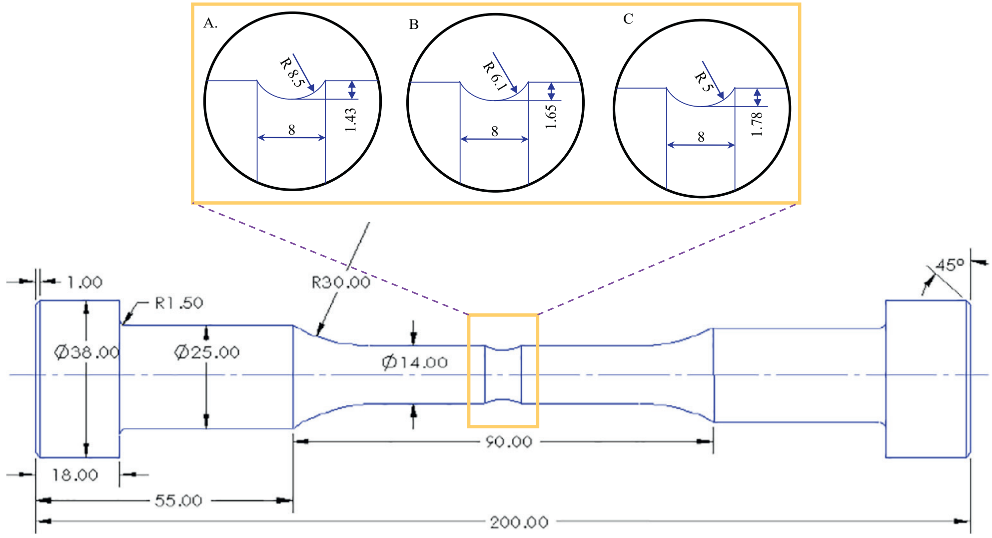

Fig. 5. Dimensions and details of notch tip parameters.

meshing elements of the notch specimen and Fig. 4(b) for the stress/strain distribution near the notch tip by elastic FE modeling. Note that the stress-strain response varies with the distance from the notch tip to the center. The geometry, dimensions and notch parameters of the notched specimens are given in Fig. 5, which are manufactured from 304 stainless steel, 40Cr steel and Ti-6Al-4V alloy [2].

Then, a new method for notch tip stress calculation is proposed as follows, by considering the size of CPZ in the notch depth, notch radius and effective area, as indicated in Fig. 3.

For the notched specimen given in Fig. 5, the notch tip stress can be expressed by a function as follow:

$$\sigma_{max} = f(\sigma, \Delta K, \sigma_y, a, R) \tag{11}$$

The geometry factor and parameters for the specimen is indicated in Fig. 6:

$$\Delta K = F\sigma\sqrt{\pi a} \tag{12}$$

$$F = \frac{1}{2\beta^{1.5}}\left[1+\frac{1}{2}\beta+\frac{3}{8}\beta^2-0.363\beta^3+0.731\beta^4\right] \tag{13}$$

where F is the geometry factor [8].

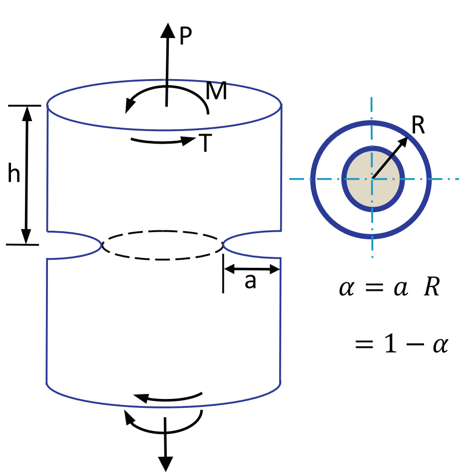

Fig. 6. Geometry factor and parameters for round bar circumferential crack (notch).

In this analysis, damaged notch depth to nominal radius ratio p is defined by

$$p = \frac{a+2r'_y}{R} \tag{14}$$

Moreover, damaged notch depth to damaged notch radius factor g is formulated as

$$g = \frac{a+2r'_y}{r-2r'_y} \tag{15}$$

Notch effect factor μ can be given by

$$\mu = (1+pg) \tag{16}$$

Crack (notch) extension coefficient δ is:

$$\delta = \frac{a}{a+2r'_y} \tag{17}$$

Undamaged to damaged area radius ratio λ is:

$$\lambda = \frac{R-a}{R-a-2r'_y} \tag{18}$$

Table 2
Mechanical properties of the three materials [2].

Material	σ_y (MPa)	σ_u (MPa)	E (GPa)
Ti-6Al-4V alloy	948	1017	118
40Cr steel	805	915	202
304 stainless steel	310	577	201

Table 3
Fatigue properties of 304 stainless steel [2].

ε_f'	c	σ_f' (MPa)	b	n'	K' (MPa)	E (GPa)
0.096	−0.446	798	−0.055	0.12332	1065.4	201

Table 4
Experimental lives of the three materials [2].

Material	σ_n (MPa)	199	178	167	158
Ti-6Al-4V alloy	K_t=1.32	5792	10,593	20,713	33,665
	K_t=1.37	4255	7652	22,793	21,413
	K_t=1.46	2917	6935	7071	22,467
40Cr steel	K_t=1.32	4350	11,623	19,785	33,082
	K_t=1.37	3829	6886	23,866	21,109
	K_t=1.46	2624	7083	6067	23,029
304 stainless steel	K_t=1.32	4346	7597	14,575	24,445
	K_t=1.37	2715	5182	15,528	15,020
	K_t=1.46	1799	4653	4692	14,429

Through combining Eqs. (12)–(18), the notch tip stress can be formulated as:

$$\sigma_{xa} = \frac{\sigma R^3}{\left(R - a - 2r'_y\right)^3}[\mu \cdot \delta \cdot \lambda] \quad (19)$$

By recalling and rearranging Massing's hypothesis (Ramberg-Osgood) in Eq. (20) and let.

$$\sigma_t = E\Delta\varepsilon_t \text{ and } \frac{\Delta\varepsilon_t}{2} = \frac{\Delta\sigma_x}{2E} + \left(\frac{\Delta\sigma_x}{2K'}\right)^{\frac{1}{n'}} \quad (20)$$

with the correlations

$$n' = \frac{b}{c} \text{ and } K' = \frac{\sigma'_f}{\left(\varepsilon'_f\right)^{\frac{b}{c}}} \quad (21)$$

$$\sigma_t = \Delta\sigma_x + 2E\left(\frac{\Delta\sigma_x}{K'}\right)^{\frac{1}{n'}} \quad (22)$$

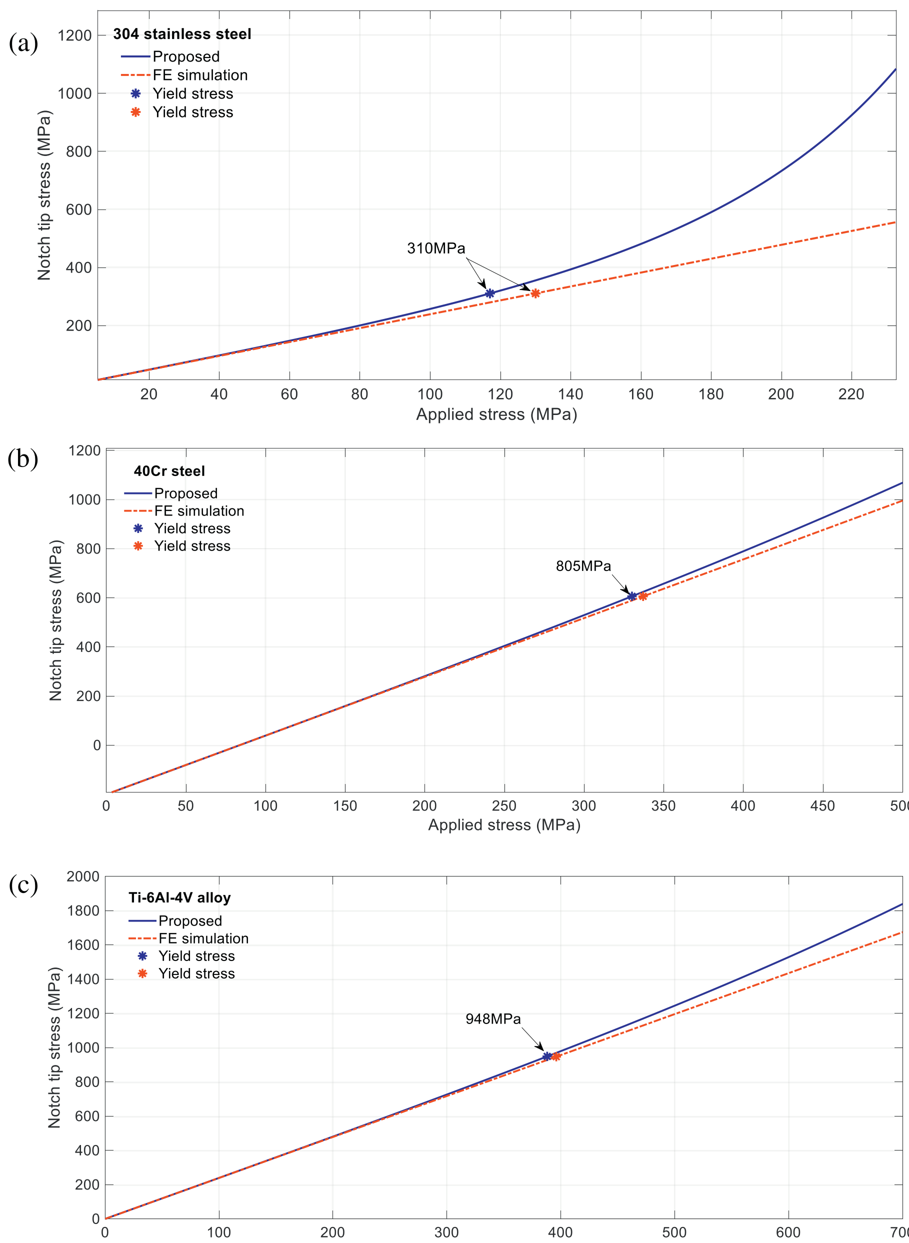

Fig. 7. Comparison of notch tip stress calculation between FEA result and proposed model predictions for (a) 304 stainless steel (b) 40Cr steel and (c) Ti-6Al-4V alloy.

Substituting Eq. (21) into Eq. (7), it yields:

$$N_f = \frac{1}{\beta+1}\cdot\frac{1}{1-\alpha}M_o^{\beta}[\sigma_t]^{-\beta} \tag{23}$$

Let $q = \frac{1}{\beta+1}\cdot\frac{1}{1-\alpha}M_o^{\beta}$, then a new fatigue life prediction model based on the CPZ and damage evolution process can be derived as follows:

$$N_f = q\left[\frac{2\cdot\sigma R^3}{\left(R-a-2r'_y\right)^3}[\mu\cdot\delta\cdot\lambda] + 2E\left(\frac{\sigma R^3}{K'\left(R-a-2r'_y\right)^3}[\mu\cdot\delta\cdot\lambda]\right)^{\frac{1}{n'}}\right]^{-\beta} \tag{24}$$

4. Model validation and comparison

4.1. Experiments

In this section, experimental data of three materials [2], namely Ti-6Al-4V alloy, 40Cr steel and 304 stainless steel, are collected for model validation and comparison with Manson-Coffin equation and Liu's model. Tables 2 and 3 list the mechanical properties of the three materials. The specimens were processed according to ASTM E606-92, GB/T15248-94 and GB3075-82 standards. Stress controlled uniaxial fatigue tests are conducted at room temperature. Moreover, the loading path is triangular waveform with stress ratio $R= -1$ and 1 Hz frequency. Experimental results are listed in Table 4.

4.2. Results and discussions

By coupling the concept of CPZ for notch fatigue analysis, a new notch tip stress calculation model is formulated by considering the size of CPZ in the notch depth, notch radius and the cross sectional area of specimen. Fig. 7 plots the size of CPZ for different applied loads with the stress response of notched specimens. For the 304 stainless steel, the CPZ size is larger than that of 40Cr steel and Ti-6Al-4V alloy.

For the proposed model, Eq. (19) estimates the notch tip stress in the direction of the applied load under cyclic loadings. However, comparing with FEA results, it clearly shows the effect of CPZ on the notch tip stress. Fig. 7 shows a comparison between the proposed model predictions and FEA calculations for the three materials, respectively. The blue and red stars indicate the point at which the notch tip stress reach the yield stress of the material. Meanwhile, the predicted values using the proposed model is almost the same with FEA results till the notch tip stress close by and pass the material yield point. As seen from Fig. 7(a), under the same load, the effect and size of CPZ for 304 stainless steel is relatively larger than that of another two materials as indicated in Fig. 7(b) and (c). By comparing results in Fig. 7, it's worth noting that due to large plasticity damage induced at the notch tip, the notch tip stress reaches the material yield point earlier than the FE simulation. Under cyclic loadings, for materials with lower yield stress, the plasticity damage greatly affect the stress build up at the notch tip.

The proposed model verifies that the notch tip stress is closely related to the material strength. Since 304 stainless steel owns relatively lower yield stress, its damaged CPZ size is larger than that of another two materials. In addition, the notch tip stress shows more than 10% variation with FE results as shown in Fig. 7. Moreover, plasticity increases drastically if the stress induced in material is close-by and beyond yield points for those materials with lower yield stress. However, for 40Cr steel and Ti-6Al-4V alloy, the variation is less than 5% till it exceeds their yield points as shown in Fig. 7.

By using Mansson-Coffin equation, Liu's model and proposed model in Eq. (24), a comparison of predicted fatigue lives of the three models and experimental lives are shown in Fig. 8. It's worth noting that fatigue lives of the three materials are predicted within the ±1.5 scatter band of experimental results.

In order to quantitatively distinguish the prediction capabilities of the three models, an error index, namely model prediction error E_r, is introduced to evaluate the deviation between evaluated and tested results [6,21,64].

$$E_r = \ln N_{fp} - \ln N_{ft} \tag{25}$$

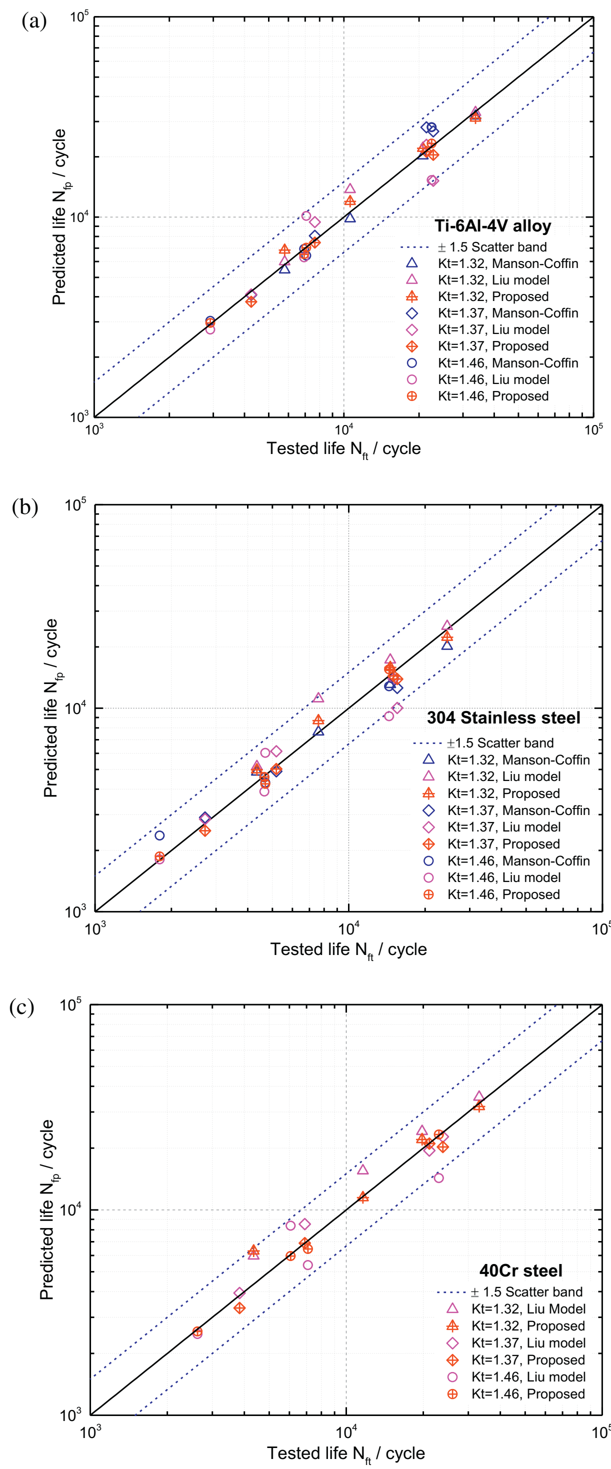

Fig. 8. Comparison of model predictions with experimental lives for (a) Ti-4Al-6 V alloy, (b) 304 stainless steel and (c) 40Cr steel.

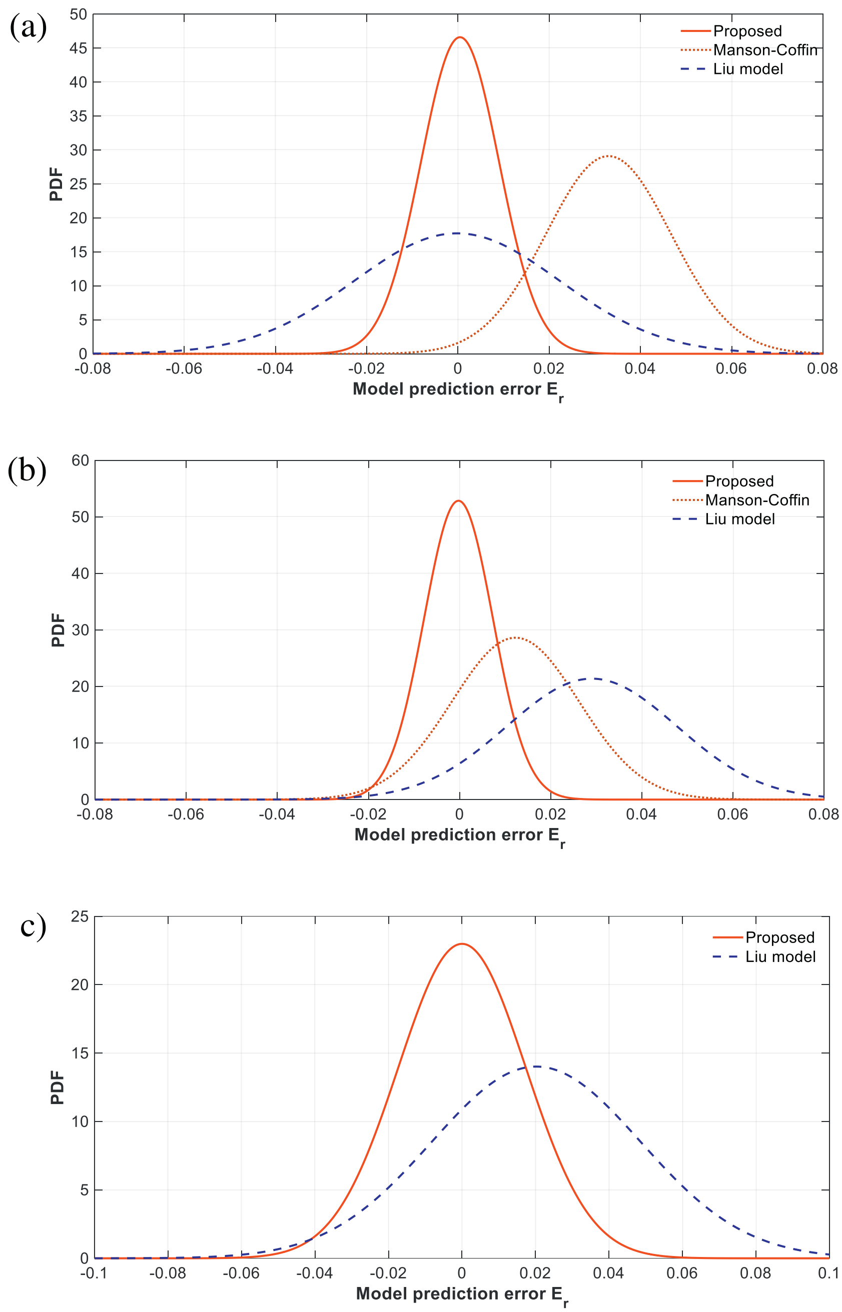

Fig. 9. Comparison of model prediction errors for (a) Ti-4Al-6 V alloy, (b) 304 stainless steel and (c) 40Cr steel.

$$E_{rm} = \frac{1}{m}\sum E_{ri} \tag{26}$$

where N_{fp} and N_{ft} are model predicted and tested lives, respectively, E_{ri} is the deviation for the ith data, E_{rm} is the mean prediction error. A model prediction is viewed as conservative when the error in Eq. (26) is negative and nonconservative for a positive one, the lower mean and standard deviation of model prediction errors correspond to higher model prediction accuracy. Model prediction errors of the three materials are plotted in Fig. 9.

Notice from Fig. 9, the proposed model yields lower mean and standard deviations of E_r, namely, it provides better life predictions compared with Manson-Coffin equation and Liu's model. Results confirm that all predictions made by the proposed model are more acceptable according to experimental fatigue lives. The proposed model in Eq. (19) can be applied to locate stress concentration regions of engineering components during closed form design and structural analysis, and Eq. (24) for notch fatigue life prediction. However, more experiments on notched specimens with different scales are desired to further verify the robustness of the proposed model for notch fatigue analysis.

5. Conclusions

In this study, the influence of CPZ on the notch tip stress and fatigue life prediction of metals are investigated based on damage evolution model. Experimental results of 304 stainless steel, 40Cr steel and Ti-6Al-4V alloy are utilized for model validation and comparison, and the following conclusions can be drawn:

1) Using concepts of CPZ and damage mechanics, a new model for determining closed-form stress at the notch tip is proposed by

considering the CPZ in terms of notch depth, notch radius and effective area.

2) The proposed model can accurately estimate the notch tip stress in the direction of applied loads, and CPZ increases the notch tip stress by altering the notch geometry due to the effect of damaged CPZ. Thus, notch tip stress, beside the geometry and size of the notch, is greatly dependent on the material strength, i.e. the CPZ size is larger for the material with lower yield stress and vice versa.
3) Fatigue life prediction of notched specimens of the three materials have been conducted combining damage evolution model with the notch tip stress calculation. Results indicate that the proposed solution yields better fatigue life predictions of the three materials than other two models.

Data availability statement

All materials data for model validation used during the study are available from the corresponding author by request.

CRediT authorship contribution statement

Anteneh Tilahun Taddesse: Conceptualization, Methodology, Writing - original draft. **Shun-Peng Zhu:** Methodology, Supervision. **Ding Liao:** Visualization, Validation, Writing - review & editing. **Behrooz Keshtegar:** Software, Writing - review & editing.

Declaration of competing interest

The authors declared that they do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted, entitled "*Cyclic plastic zone-based notch analysis and damage evolution model for fatigue life prediction of metals*".

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