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Vortex-induced vibration of a flexible fluid-conveying riser due to vessel motion

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ABSTRACT

As vessel motion and internal flow can exert impact on vortex-induced vibration (VIV) of flexible risers, it is of great significance to take these two factors into consideration while VIV is investigated. Therefore, crossflow (CF) VIV of a flexible fluid-conveying riser is studied when vessel motion is considered. The governing equation is firstly discretized based on finite element method and solved by Newmark-β method. Then, validation is conducted by comparing with experimental data. Subsequently, CF VIV response of a flexible riser is explored with consideration of both vessel motion and internal flow. The results show that regardless of low and high Keulegan-Carpenter (KC) numbers, vessel motion can have a notable effect on top tension and CF VIV characteristics. Variations of CF VIV amplitude, vibration frequency as well as top tension frequency are affected by both vessel motion and internal flow. With the increase of internal flow velocity, CF VIV amplitude is changed while mode jump occurs concomitant with frequency transition. Besides, both vessel motion and internal flow velocity can have an impact on energy allocation along the flexible fluid-conveying riser. Noteworthy is that although both vessel motion and internal flow can exert an impact on top tension and VIV response, the effect induced by vessel motion still plays a dominated role.

1. Introduction

During deep sea resource exploitation, risers are widely utilized to transport the mixture of gas and oil as well as ores from seabed to sea surface, indicating the significant role of riser systems in ocean engineering industry. Since the environmental conditions in the ocean are extremely complicated during the exploitation of deep sea resources, it is inevitable for these riser systems to undergo external loads caused by various oceanic factors, such as the coupling of external wind, wave, current and vessel motion, the hydrodynamic force of the internal waves as well as the effect of internal and external flows. As a result, the safety of equipment applied in ocean engineering can be endangered, thereby threatening deep sea resource exploitation. Note that among these structural dynamics, vortex-induced vibration (VIV) of flexible risers due to complicated oceanic environmental conditions is pivotal as a nonignorable factor. Therefore, VIV mechanism and characteristics have been massively investigated by many researchers so that solid foundations can be provided for riser design and analysis in ocean engineering [1–3].

Since VIV of pipes or risers became an issue worthy further attention for ocean resource exploitation, continuous efforts have been made to the investigations of VIV mechanism and characteristics, experimentally and numerically. In these researches, VIV dynamics of rigid cylinders is firstly focused and studied, including free and forced vibrations of rigid cylinders. And the structural response and hydrodynamics of these rigid cylinders are mainly detected and analyzed. For example, through the experimental study carried out by Feng [4], two response amplitude branches (initial and lower branches) were observed for the cylinders with high mass ratios. Whereas three response amplitude branches (initial, upper and lower branches) for the cylinders with low mass ratios were detected with the increase of the reduced velocity in the work of Khalak and Williamson [5]. Besides, different vortex shedding patterns due to the cylinder motion, such as 2S, 2P, P+S and 2S+2P, were also found in the wake field behind these rigid cylinders by Khalak and Williamson [6] and Singh and Mittal [7]. When the vortex shedding frequency is entrained to coincide with the structural vibration frequency, the occurrence of lock-in is captured [8], wherein typical VIV features, such as self-limited response amplitudes and a wide range of

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synchronization frequencies, can be evidently observed [9]. All these findings have provided us with basic knowledge for the future research of VIV.

As resource exploitation moves towards deep sea, marine risers widely applied to transport resource tend to become flexible. Therefore, concentrations on VIV of flexible risers have been increasing nowadays. Correspondingly, experiments have been carried out by many investigators so that VIV response of flexible risers can be explored [10–17]. Among the studies of Trim et al. [8] and Huera-Huarte et al. [9], multi-mode VIV was captured rather than single mode VIV due to large ratio of length to diameter, which was concomitant with more complicated VIV characteristics. Meanwhile, hydrodynamic characteristics were also examined through experiments by Han et al. [9] and Brika and Laneville [17], wherein added mass, drag and lift coefficients were mainly analyzed and discussed. Apart from experimental researches, by using computational fluid dynamics (CFD) and developing semi-empirical models, such as wake oscillator model, VIV response of flexible risers under various currents has also been explored [18–23]. For instance, the thick strip method has been utilized by Bao et al. [21] to predict VIV of flexible risers under low Reynolds number. Instead of two wake oscillators, a one-wake-oscillator model was developed by Qu and Metrikine [22,23] to study VIV of flexible risers numerically, which showed a good agreement with experimental data. By performing numerical works, the vibration amplitude, frequency and excitation modes in both inline (IL) and crossflow (CF) directions [19,24], as well as fatigue damage of the flexible riser [25], were also analyzed and discussed when the flexible riser was subjected to uniform and shear flows. Moreover, VIV response considering the change of top tension was also investigated by Yuan et al. [26], Wang et al. [27] and Li et al. [28], which proves that the top tension can have a notable effect on the bending stiffness of risers. As oscillatory flow can be generated due to the relative motion between the riser and fluid, VIV mechanism and characteristics of cylinders undergoing oscillatory flow have also been focused by many researchers [29–36]. It is found experimentally by Fu et al. [29], Resvanis [30], Wang et al. [31,32] and Fan et al. [33] that more complicated VIV characteristics can be induced under oscillatory flow, such as intermittent VIV, hysteresis, mode jump, frequency transition and amplitude modulation. And these typical VIV features under oscillatory flow were also numerically captured by Thorsen et al. [34], Yuan et al. [35] and Srinil et al. [36]. Apparently, more thorough and comprehensive understandings of VIV have been obtained based on these investigations while various types of external flow are considered.

Note that although VIV of risers undergoing uniform, shear and oscillatory flows has been explored by many researchers, these studies are mainly carried out by simplifying the external environmental conditions. As far as we are concerned, the riser system is connected with the platform on the sea surface. Due to the coupling of wind, wave and current, there exist dynamic motions of the platforms, which can affect VIV response of flexible risers during the exploitation of deep sea resource. Generally, the oscillatory vessel motions are usually referred to as surge, sway and heave, with heave being the vertical motion. Surge or sway motion is mainly caused by the nonlinear effect of wave and wind, with typical period of the order of one to two minutes. Heave is an important response variable for many structures, the natural period of which depends on different types of structures. As the motion of vessel can induce complicated VIV that aggravates riser damage, VIV response of flexible risers with consideration of vessel motion needs more attention. Correspondingly, some studies on VIV of risers have emerged while vessel motion is taken into account. Through conducting experiments, VIV of a free-hanging riser or a catenary riser has been studied by Wang et al. [37,38] and Mao et al. [39], in which complicated VIV response was observed due to vessel motion. And the fatigue damage caused by vessel motion was found serious. Besides, numerical works have also been carried out by Chen et al. [40] and Yuan et al. [41] while the effect of the amplitude and frequency of top-end vessel sway on flexible risers was taken into consideration. It is found that complicated VIV features, such as multimode participation, travelling wave, amplitude modulation and time-sharing phenomenon were directly captured under vessel motion. Furthermore, the VIV displacements of flexible risers were found related to surge or heave motion by Fan et al. [42], Guo et al. [43] and Chen et al. [44], indicating the unignorable effect of vessel motion on VIV. All these studies have proved that vessel motion can exert an impact on VIV, which should not be ignored.

It should be emphasized that all the aforesaid researches focus on VIV of risers without internal flow. Nevertheless, internal flow has been proved to have an influence on VIV response by many researchers [45,46]. For instance, the natural frequency of the risers or pipes was experimentally found to decrease with the increase of the internal flow velocity by Guo et al. [47] and Zhu et al. [48,49], which can affect the lock-in domain as well as mode jump and frequency transition of VIV. In addition, regardless of external flow, VIV characteristics, such as vibration amplitude, frequency, standing and travelling wave responses, were also changed with the increase of internal flow velocity, which has been studied by Meng et al. [50], Wang et al. [51], Yang et al. [52], Jiang et al. [53] and Duan et al. [54]. Moreover, the effect of internal flow density on VIV has been studied numerically by many researchers, in which the excited mode response and vibration frequency have been proved closely related to the change of internal flow density [54–57]. Therefore, it can be concluded that while VIV response of a flexible riser is studied, the influence of internal flow should be considered.

Since either vessel motion or internal flow plays an important role in the occurrence of VIV, it is necessary to consider both factors during the investigation of VIV. What should also be noticed is that the coupling mechanism between different types of vessel motion and VIV is essentially distinct. When heave occurs, an additional periodical excitation is induced and exerted on the riser top vertically, which can have a periodical effect on structural property. While there exists surge or sway motion of vessel, transverse vibration along the riser can be triggered, which will propagate from riser top to end. In addition, vibrating boundary due to vessel motion can lead to complicatedly nonlinear responses while coupling with VIV. Due to these vessel motions, VIV response will be undoubtedly nonlinearized and complicated. However, it is extremely challenging to take into account all these types of vessel motion while VIV is studied. Therefore, only surge motion is considered while VIV of a flexible fluid-conveying riser is investigated in our work. It can be said that the main novelty of our work lies in the investigation on VIV response while both surge motion of vessel and internal flow are taken into consideration. Hence, the main objective of the present work is to explore CF VIV of a flexible riser while vessel motion as well as internal flow is considered. Correspondingly, the effect of vessel motion and internal flow velocity on VIV characteristics in CF direction is examined and discussed, including variation of top tension, amplitude and frequency. And the paper is structured as follows. In Section 2, the applied model is explained in detail, including the structural model and hydrodynamic force model. Then validation is performed based on the existing experimental data in Section 3 so that the feasibility and accuracy of the numerical method can be examined. Subsequently, CF VIV response of a flexible fluid-conveying riser is studied while the surge motion of vessel is considered in Section 4. Correspondingly, the VIV mechanism and characteristics are mainly analyzed and discussed with different Keulegan-Carpenter (KC) numbers and nondimensional internal flow velocities. Finally, conclusions are drawn in Section 5.

2. Analytical method

2.1. Governing equations

The flexible riser with consideration of vessel motion and internal flow and its finite element model are illustrated in Fig. 1. It can be seen that the riser top is connected with the vessel while the connection between the riser bottom and seabed is modelled by a hinge. And only the surge motion of vessel is taken into account during our simulation. It

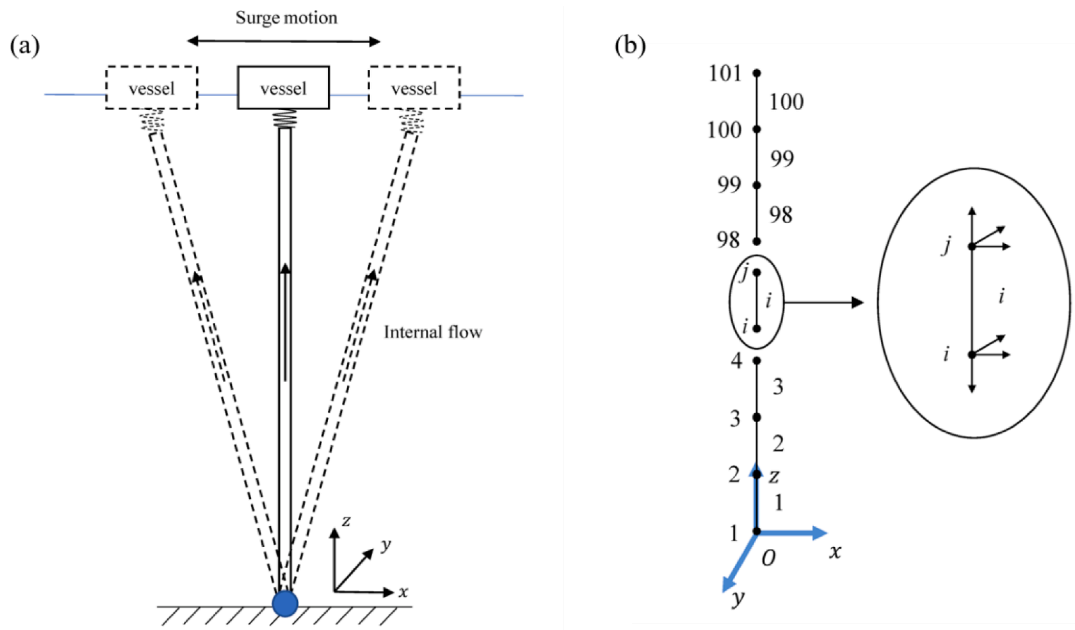

Fig. 1. Schematic of (a) a fluid-conveying riser with consideration of vessel motion and (b) finite element model of the flexible riser.

should be noticed that when the surge motion is excited, concomitant oscillation of the riser can be induced. Besides, the constraint bottom of the riser causes that the vibration displacement of the riser decreases from top to end. Similar to the works of Yuan et al. [41] and Xue et al. [64], the encountered flow field of the riser considering vessel motion and internal flow is simplified to a combined sheared-oscillatory flow, which is non-uniform in both time and spatial domains. Correspondingly, considering the gravity and buoyancy forces as well as the effect of top tension variation, the governing equations of CF VIV of the flexible fluid-conveying riser can be expressed as [58]:

$$\left(m_r+m_f+m_a\right)\frac{\partial^2 y}{\partial t^2}+c\frac{\partial y}{\partial t}+2m_fU_i\frac{\partial^2 y}{\partial z\partial t}+EI\frac{\partial^4 y(z,t)}{\partial z^4}+\left[\left(m_r+m_f+m_a\right)g-F_b\right]\frac{\partial y}{\partial z}+\left(m_fU_i^2-T\right)\frac{\partial^2 y}{\partial z^2}-\left(m_r+m_f+m_a\right)g(L-z)\frac{\partial^2 y}{\partial z^2}=F_{CF} \tag{1}$$

where y denotes the dimensional displacement in CF direction, m_r, m_f and m_a are the flexible riser mass, internal flow mass and added mass per unit length respectively. m_a can be obtained based on $m_a = C_a\rho\pi D^2/4$, where ρ and D denote external flow density and diameter respectively. c represents the structural damping. U_i is the internal flow velocity. F_b represents the buoyancy force per unit length. EI and L are the bending stiffness and riser length respectively. T denotes the top tension, which changes with the vibration of the flexible riser. g is the gravitational acceleration. F_{CF} represents the CF hydrodynamic force per unit riser length caused by the vessel surge motion, including hydrodynamic excitation and damping forces. Note that Rayleigh damping model is adopted to calculate c. According to Païdoussis and Issid [58], the effect of internal flow on CF VIV is mainly represented by the third and sixth terms of the governing equation. The third term denotes that the Coriolis force caused by the internal fluid can exert an excitation or damping effect on VIV under different conditions. While the sixth term in the governing equation indicates that the centrifugal force induced by the internal flow can change the stiffness of the flexible fluid-conveying riser. Therefore, the effect of internal flow on VIV response is taken into account in the model.

Since the top tension of the flexible fluid-conveying riser is time-varying, it needs to be calculated at each timestep. And the variable top tension can be obtained based on the following equation:

$$T = T_0 + EA_r\frac{S-L}{L} \tag{2}$$

where T_0 is the initial value of top tension, E represents the Young's modulus of elasticity, A_r denotes the cross section of the flexible fluid-conveying riser. S and L are the values of deflected and initial riser lengths respectively. And S can be calculated based on $S = L\sqrt{1+\left(\frac{\partial y}{\partial z}\right)^2}$ when only CF VIV is considered and investigated.

2.2. External hydrodynamic force

The external hydrodynamic force acting on the flexible fluid-conveying riser (per unit riser length) mainly includes excitation force and damping force, which is as follows:

$$F_{hydro} = F_{ex} + F_{damp} = \frac{1}{2}\rho DU_e^2C_L\cos(\omega\cdot t+\varphi) - \frac{1}{2}\rho DC_{damp}|\dot{y}|\dot{y} \tag{3}$$

where ρ represents the external flow density, D is the external diameter of the flexible riser, U_e denotes the external flow velocity, C_L and C_{damp} represent the hydrodynamic excitation coefficient and hydrodynamic damping coefficient, ω and ϕ denote the CF vibration frequency and phase angle. $\dot{y}$ represents the oscillation velocity of the flexible riser in CF direction. During the calculation of the external hydrodynamic force, the values of C_L and C_{damp} need to be determined. The identification of excitation and damping regions as well as criterion for the selection of the excitation hydrodynamic coefficient C_L have been elaborated in the works of Duan et al. [54,57] and Zhang et al. [59]. Therefore, the detailed process will not be interpreted here. After the corrective values of C_L are determined at each timestep, the corresponding hydrodynamic excitation force is obtained, and then exerted on the excitation region of the flexible fluid-conveying riser.

As for the hydrodynamic damping coefficient, $C_{damp} = a_0 + a_1 y/D$ is adopted here based on the investigation of VIV under oscillatory flow by Thorsen et al. [34]. In their study, it is proved that the model with $a_0 = 0.31$ and $a_1 = 0.89$ can yield a good approximation of the hydrodynamic damping force. And the numerical results there show a good agreement with experimental data. Therefore, $C_{damp} = 0.31 + 0.89y/D$ is applied during our simulation. Correspondingly, the hydrodynamic damping force exerted on the damping region of the flexible fluid-conveying riser can be calculated.

After the excitation and damping regions of the flexible riser are identified, the CF hydrodynamic excitation and damping forces are calculated and exerted on the flexible fluid-conveying riser. By coupling with the structural model, CF VIV response of the flexible riser can be obtained while both vessel motion and internal flow are taken into account.

2.3. Method of analysis

The finite element method (FEM) is applied to discretize the governing equations. Then the differential equation of motion for the flexible fluid-conveying riser is solved by using Newmark-β method. According to Huebner [60], the motion equation of the flexible riser in CF direction can be given as

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{f(t)\} \tag{4}$$

where $[M]$ denotes the riser mass matrix, $[C]$ the riser damping matrix and $[K]$ the riser stiffness matrix. $\{\ddot{y}\}$, $\{\dot{y}\}$ and $\{y\}$ are the vectors of CF VIV acceleration, velocity and displacement, respectively. $\{f(t)\}$ represents the vector of external hydrodynamic forces.

During discretization, the piecewise cubic Hermite interpolation is used. Correspondingly, the element mass matrix can be obtained by integrating

$$[M]^e = \int_0^l N^T (m_r + m_f + m_a) N dz \tag{5}$$

$$[C]^e = \int_0^l N^T c N dz - \int_0^l N^T 2 m_f U_i \left(\frac{\partial N}{\partial z}\right) dz \tag{6}$$

where N is the shape function, l represents the length of the riser element. And the element stiffness matrix $[K]^e$ consists of matrixes related to geometry, bending stiffness, internal flow, gravity and buoyancy force, which can be expressed as

$$[K]^e = [K]^{e1} + [K]^{e2} + [K]^{e3} + [K]^{e4} \tag{7}$$

in which

$$[K]^{e1} = \int_0^l \left(\frac{\partial N}{\partial z}\right)^T \left(m_f U_i^2 - T\right) \left(\frac{\partial N}{\partial z}\right) dz \tag{8}$$

$$[K]^{e2} = \int_0^l N^T \left[-(m_r + m_f + m_a) g (L - z) \right] \left(\frac{\partial^2 N}{\partial z^2}\right) dz \tag{9}$$

$$[K]^{e3} = \int_0^l \left(\frac{\partial^2 N}{\partial z^2}\right)^T EI \left(\frac{\partial^2 N}{\partial z^2}\right) dz \tag{10}$$

$$[K]^{e4} = \int_0^l N^T \left[(m_r + m_f + m_a) g - F_b \right] \left(\frac{\partial N}{\partial z}\right) dz \tag{11}$$

After the governing equation is discretized based on FEM, the derived differential equation can be solved through Newmark-β method. Then VIV response of the flexible fluid-conveying riser in CF direction can be obtained with consideration of vessel motion and internal flow.

3. Validation against experimental data

The adopted model has been validated based on the comparison of experimental and numerical results and utilized for simulating VIV response of a flexible fluid-conveying riser undergoing uniform and shear currents by Duan et al. [54,57] respectively, which has proved the feasibility and accuracy of the prediction model on VIV. Generally, it is believed that the adopted numerical model can capture reasonable prediction results of VIV under external uniform and shear currents. Nevertheless, since few experiments can be found for VIV of a flexible riser with internal flow, validation continues to be carried out with consideration of only vessel motion here. The experiments carried out by Yin et al. [61] are chosen here. Detailed information about the experiment can be obtained from the work of Yin et al. [61].

Before the implement of our validation, some important parameters for VIV undergoing vessel motion need to be introduced. The surge motion of vessel at the flexible riser top is assumed to be simple harmonic oscillation, which can be described as

$$A(t) = A_m \sin\left(\frac{2\pi}{T_m} t\right) \tag{12}$$

where A_m denotes the maximal amplitude of surge motion, T_m is the oscillation period of vessel motion. Correspondingly, the oscillation velocity can be expressed as

$$V(t) = A_m \frac{2\pi}{T_m} \cos\left(\frac{2\pi}{T_m} t\right) = V_m \cos\left(\frac{2\pi}{T_m} t\right) \tag{13}$$

where V_m represents the maximal oscillation velocity of the riser top due to vessel motion.

Then validation continues to be carried out by comparing with experimental data from Yin et al. [61]. The corresponding riser model parameters are listed in Table 1. And the tension here refers to the one measured from the riser top.

Note that the damping ratio is not given here. The values for the damping ratio of first and second mode responses are set to equal to 0.01 here based on the confirmation from the experiment conductors. Then the damping matrix is obtained based on Rayleigh damping model, parameters of which can be obtained by using natural frequencies of the first and second dominant modes. Another parameter worthy attention is added mass coefficient C_a. As is stated, when the external flow is unsteady, the value of added mass coefficient C_a is not constant and can be affected by many factors like Reynolds number, reduced velocity and KC number [62]. And the model with $C_a = 1.5$ can yield reasonably correct numerical results compared with experimental data, which can also be observed in the following discussion. Therefore, $C_a = 1.5$ is applied in our work. Then three cases are chosen to simulate for validation, as shown in Table 2.

Fig. 2 and Table 3 present the comparison of maximal amplitude and vibration frequency of CF VIV between the numerical and experimental results. It can be seen that the numerical results show a reasonably good agreement with the experimental data. Multi-mode and multi-frequency VIV responses are detected and captured by the numerical model. As shown in Table 3, the vibration frequencies of CF VIV corresponding to first three mode response were tested in the experiment. Other than

Table 1
Parameters of the flexible riser model [61].

Property	Value
Length L (m)	8.996
Bending stiffness EI (N • m^2)	120
Axial stiffness EA_r (N)	1.79×10^6
Outer diameter D (m)	0.028
Inner diameter d (m)	0.017
Tension T (N)	212
Mass per unit length m_r (kg/m)	0.668

Table 2
Experimental cases simulated for comparison [61].

Case	A_m(m)	T_m(s)
1	0.052	0.677
2	0.052	1.547
3	0.078	1.547

these CF VIV frequencies, more oscillation frequencies in CF direction were also detected due to vessel motion by Yin et al. [61]. It can be seen from Table 3 that CF VIV frequencies related to the first three mode responses can be also captured during our simulation. Besides, more vibration frequencies are observed too due to vessel motion. Although the oscillation frequencies for CF VIV induced by vessel motion are not completely in agreement with the experimental ones, main CF VIV frequencies can be reproduced during our simulation. It should be mentioned that there exists some reasonable discrepancy for some cases. This can be attributed to the hydrodynamic excitation coefficient adopted during the simulation. Since no available hydrodynamic excitation coefficient data can be utilized for VIV of flexible risers subjected to vessel motion, the applied ones here are obtained based on forced oscillation tests of rigid cylinder by Gopalkrishnan [63]. Consequently, it is inevitable for the existence of disparity between the experiment and numerical simulation, which can also be observed in the works of Yuan et al. [35,41] and Duan et al. [54,57]. Another factor accounting for such discrepancy is that only CF VIV response is studied here without considering the coupling of IL and CF VIV during the simulation, which can lead to reasonable disparity. However, since main CF VIV characteristics can be reproduced by the proposed model, it can be concluded that the present model is feasible for the prediction of CF VIV response due to vessel motion. Therefore, by utilizing the proposed model, CF VIV response of a flexible riser is investigated in the following section while both internal flow and vessel motion are taken into account.

4. CF VIV response of a flexible fluid-conveying riser due to vessel motion

In this section, CF VIV response of a flexible fluid-conveying riser undergoing external oscillatory flow current due to vessel motion is investigated. To describe a certain oscillatory flow induced by vessel motion, KC number is defined as a key parameter [29,30]. The expression of KC number is given as follows

Table 3
Vibration frequency of experiment and simulation.

Case	Frequency (Hz) Experiment [61]	Simulation
1	0.22, 1.48, 2.95	0.52, 1.44, 2.44
2	0.65, 1.29	0.66, 1.96
3	0.18, 0.45, 0.65, 0.83, 1.28, 1.75	0.14, 0.66, 1.14, 1.44, 2.44

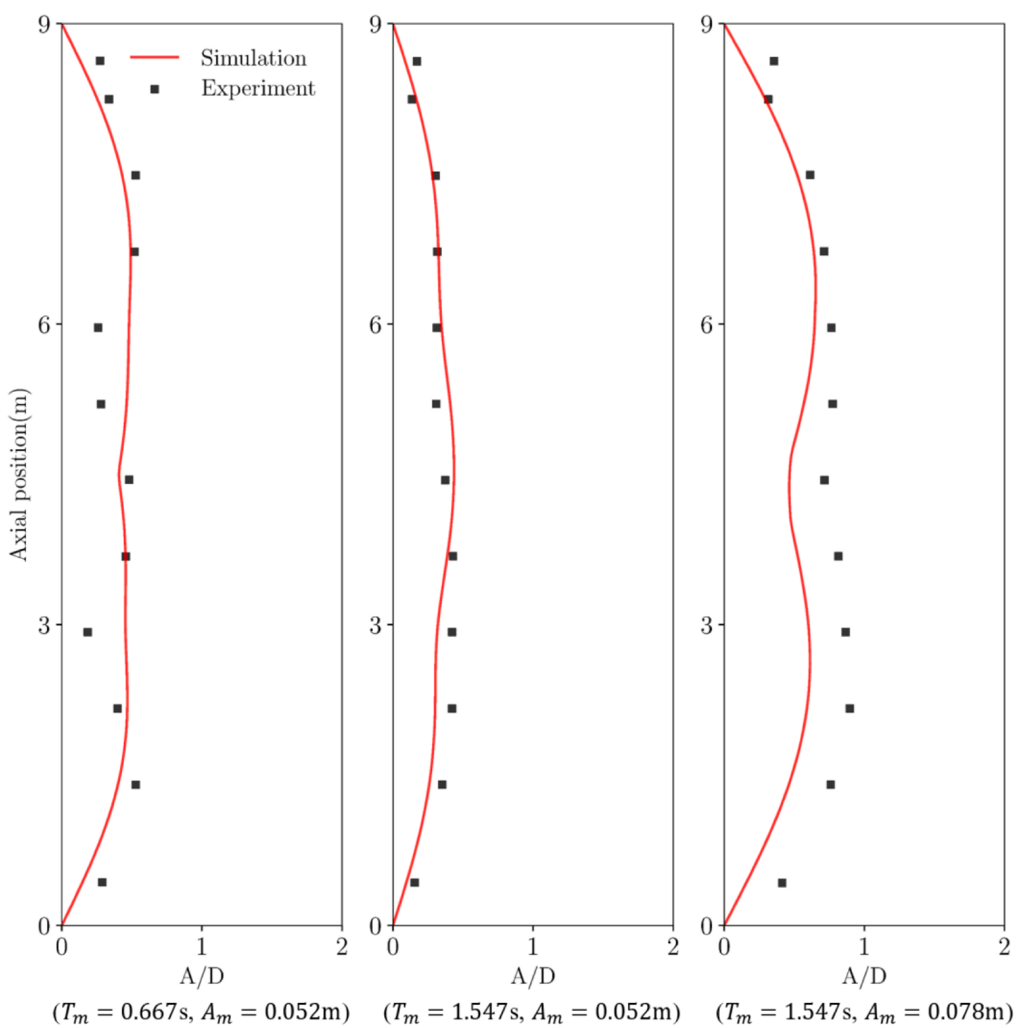

Fig. 2. Comparison of maximal CF VIV amplitude between numerical and experiment results under three different cases.

$$KC = \frac{VT_m}{D} = \frac{2\pi A}{D} \tag{14}$$

where V, T_m and A denote the velocity, period and amplitude of the oscillatory flow respectively. Since the external current velocity along the flexible riser due to vessel motion is not constant, KC number during our study denotes the KC number at the riser top, which can be calculated by using V_m or A_m (V_m and A_m are the maximal velocity and amplitude of vessel motion). Therefore, the KC number can be obtained by

$$KC = \frac{V_m T_m}{D} = \frac{2\pi A_m}{D} \tag{15}$$

The flexible riser parameters from Yin et al. [61] are still applied for the research. The internal flow velocity with density 1000kg/m^3 is nondimensionalized by $\nu = \sqrt{\frac{m_f}{EI}} \cdot LU_i$, which is set equal to 0, 1, 2, 3 and 4. Besides, high KC number 500 and low KC number 100 are chosen here with T_m = 32s and 16s respectively. Correspondingly, top tension and VIV characteristics in CF direction, such as vibration amplitude and frequency, are mainly analyzed and discussed while both vessel motion and internal flow are considered.

As it is proved that the natural frequency of the flexible riser plays an important role in affecting VIV response, especially for the excited mode response and oscillation frequency, the variation of natural frequency is firstly examined with the increase of nondimensional internal flow velocity. Correspondingly, the changing trend of the first four natural frequencies is plotted in Fig. 3 while the internal flow velocity is increased. It can be observed that the values of natural frequency corresponding to each mode response decrease with the increase of the nondimensional internal flow velocity, which shows a similar changing trend with those in the works of Guo et al. [47], Meng et al. [50] and Duan et al. [54,57]. The decrease of natural frequency can be ascribed to the change of structural stiffness due to internal flow.

4.1. Top tension variation

When VIV occurs, the riser top tension can be changed due to riser deformation. It has been proved that such variation of top tension can exert an evident impact on VIV response by changing the riser bending stiffness [26–28], so it is necessary to focus on the change of top tension firstly while vessel motion and internal flow are taken into account. Correspondingly, the variation of riser top tension and its frequency with different nondimensional internal flow velocities for KC = 100 and 500 are depicted in Figs. 4 and 5.

4.1.1. Effect of vessel motion

Clearly, the values of top tension can be affected by vessel motion under both high and low KC numbers. During a certain period, the value of top tension modulates with the change of external flow velocity generated by vessel motion. And the occurrence of CF VIV accounts for such modulation of top tension. For both low and high KC numbers, there exist more than one frequency for variable top tension, among which vessel motion is responsible for one component of top tension frequency. It can be seen from Figs. 4 and 5 that one value of top tension frequency (approximately 0.12Hz for KC = 100and 0.06Hz for KC = 500) is related to the frequency of vessel motion (0.0625Hz for KC = 100and 0.0312Hz for KC = 500). And this branch of top tension frequency is almost twice as that of vessel motion regardless of low and high KC numbers. This can be explained by that the external flow velocity is changed periodically because of vessel motion, which leads to intermittent VIV response in CF direction. As a consequence, the tension at the riser top changes periodically due to intermittent CF VIV response.

Apart from the influence of vessel motion on top tension, the frequency of variable top tension is also notably affected by CF VIV response under low and high KC numbers. Correspondingly, there appear multi frequencies for variable top tension, particularly under the impact of intensive CF VIV response with high KC number. It can be observed from Fig. 4 and 5 that some frequencies of the variable top tension are twice as the vibration frequencies of CF VIV response. For KC = 100, the frequency of top tension equal to about 2.44Hz can be detected when $\nu = 0$, the value of which is approximately double as CF VIV frequency 1.2Hz (Figs. 4 and 9). Meanwhile, frequency values of around 2.3Hz and 4.62Hz are captured for variable top tension under KC = 500 when $\nu = 0$. And these frequencies are nearly twice as those of 1.14Hz and 2.28Hz corresponding to various mode CF VIV responses respectively (Figs. 5 and 10). Besides, top tension frequencies almost accordant to vibration frequencies of CF VIV are also observed, especially for high KC number, which can be attributed to the influence of CF VIV response on top tension. As shown in Figs. 5 and 10, the top tension frequency of around 1.2Hz is much close to CF VIV frequency 1.5Hz with KC = 500. All these findings about frequency proves the interaction between variable top tension and CF VIV response due to vessel motion.

4.1.2. Combination effect of vessel motion and internal flow

As for the effect of internal flow velocity on variable top tension while vessel motion is considered, it can be seen from Figs. 4 and 5 that

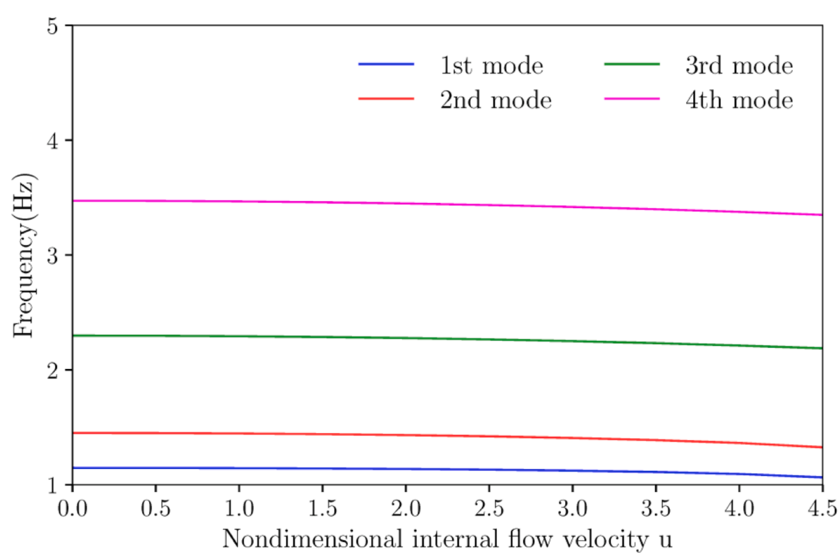

Fig. 3. Variation of first four natural frequencies with the increase of nondimensional internal flow velocity.

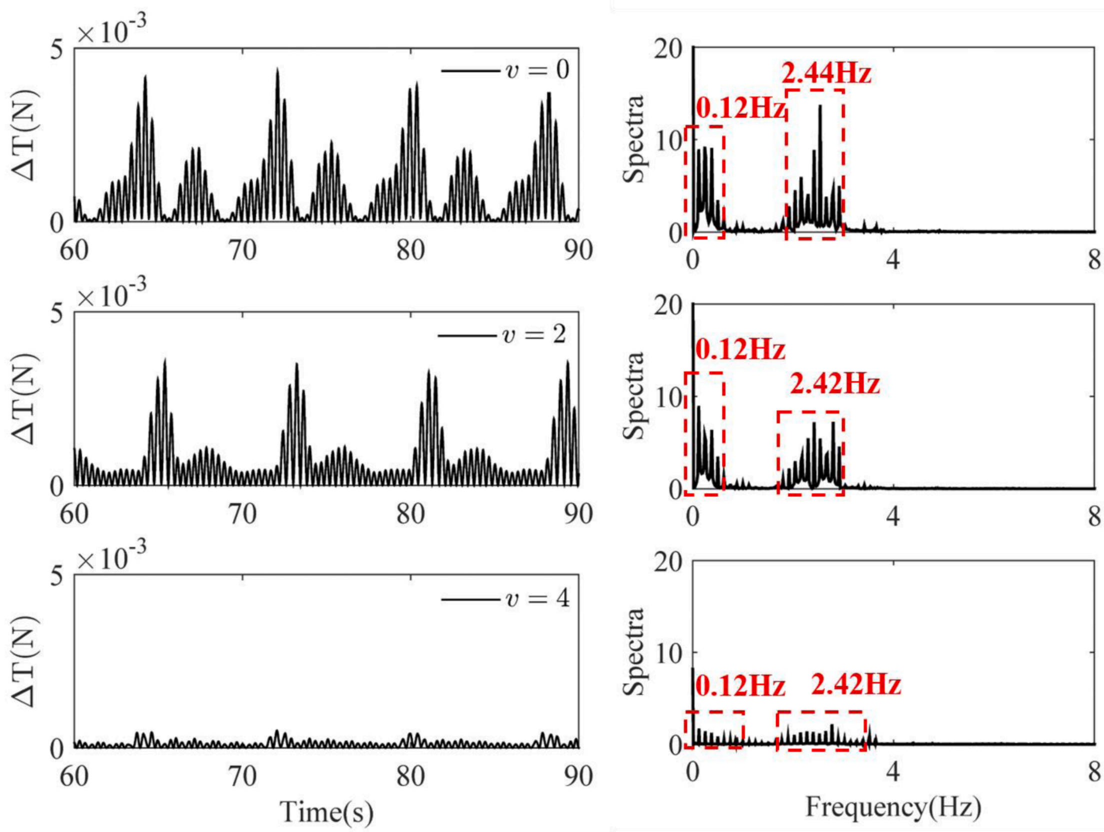

Fig. 4. Variation of top tension and its frequency with the increase of internal flow velocity under KC = 100. (The nondimensional internal flow velocity is equal to 0, 2 and 4 respectively).

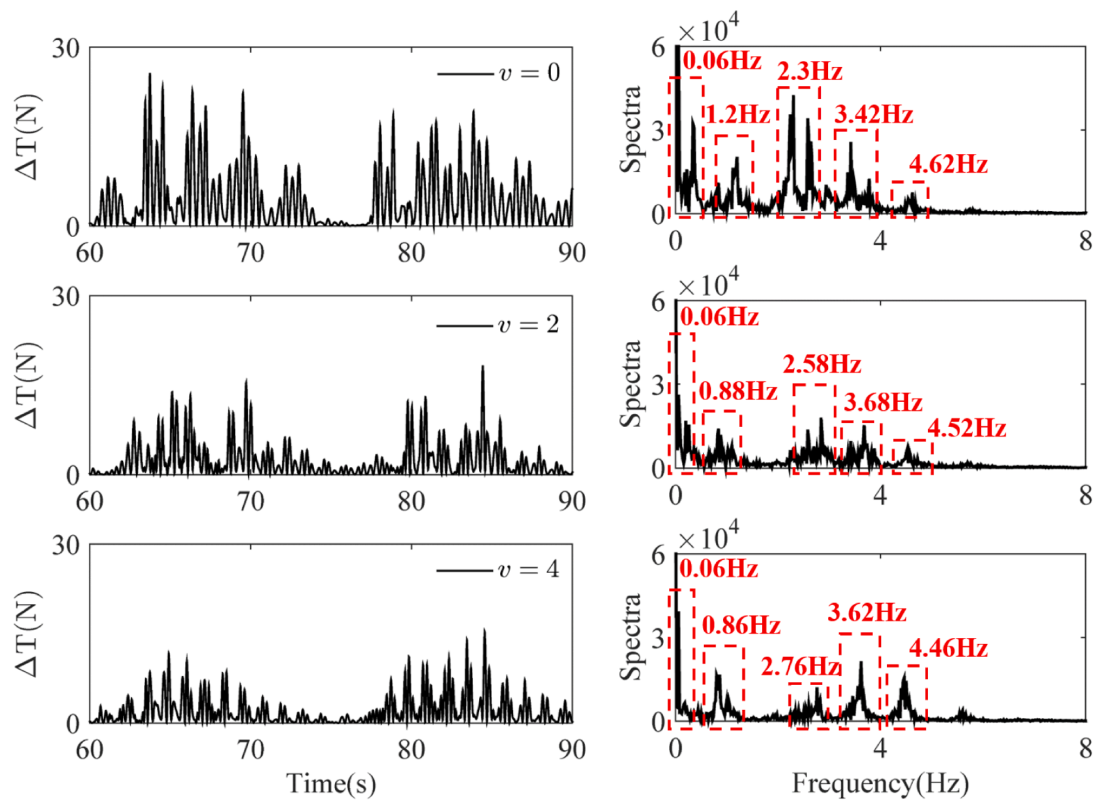

Fig. 5. Variation of top tension and its frequency with the increase of internal flow velocity under KC = 500. (The nondimensional internal flow velocity is equal to 0, 2 and 4 respectively).

the maximal value of top tension is reduced with the increase of internal flow velocity, which can be ascribed to the decrease of CF VIV amplitude caused by internal flow. Note that frequencies of variable top tension are also changed while the internal flow velocity is increased, especially for the case with high KC number. It can also be observed from in Figs. 4 and 5 that the frequency bandwidth of top tension becomes wider and its dominated frequency tends to be weaker with the increase of internal flow velocity for both low and high KC numbers. The reason for this phenomenon is that VIV response in CF direction is affected by increasing internal flow velocity, leading to the change of top tension frequency under low and high KC numbers. And the dominated frequency of top tension is weakened because of the participation of the stronger sub-harmonic components with the increase of internal flow velocity. It should be emphasized that although the effect of internal flow on top tension cannot be ignored, vessel motion still exerts a dominated impact on variable top tension of the flexible fluid-conveying riser while the internal flow velocity is increased.

4.2. CF VIV amplitude

As amplitude of VIV is an important characteristic which is related to top tension, riser strain, etc., it is significant to examine the variation of VIV amplitude while both internal flow and vessel motion are considered. Therefore, the maximal amplitude of CF VIV response of the flexible fluid-conveying riser under low and high KC numbers with different nondimensional internal flow velocities is studied and analyzed. And the results are illustrated in Fig. 6.

4.2.1. Effect of vessel motion

As shown in Fig. 6, what should be noticed is that regardless of low and high KC numbers, the maximal CF VIV amplitude along the flexible fluid-conveying riser due to vessel motion always appears in the upper section, where the induced external flow velocity due to vessel motion is large enough to trigger CF VIV response. This proves that more energy concentrates at the upper section of the flexible fluid-conveying riser. Besides, the maximal CF VIV amplitude under low KC number (KC = 100) is much smaller than those with high KC number (KC = 500) (Fig. 6), which is also observed in the work of Chen et al. [40]. This is because when the top of the flexible fluid-conveying riser moves with vessel motion, corresponding external oscillatory flow velocity is generated along the flexible riser. It means that the upper section of the flexible fluid-conveying riser is subjected to high external oscillatory flow velocity. Consequently, CF VIV response is excited on the upper section of the flexible riser, leading to large vibration amplitude. Nonetheless, since the external oscillatory flow exerted on the bottom section of the flexible riser due to vessel motion is low, the critical velocity for the occurrence of VIV cannot be achieved. As a result, CF VIV cannot be triggered on the bottom section of the riser, causing the appearance of small CF VIV amplitude, as shown in Fig. 6. Another potential factor for the appearance of large CF VIV amplitude is that relatively large hydrodynamic force is exerted on the upper section of the flexible riser, leading to more energy concentration here.

4.2.2. Combination effect of vessel motion and internal flow

Variation of CF VIV amplitude can be captured obviously while both vessel motion and internal flow are considered. It can be seen that with low KC number, the maximal VIV amplitude in CF direction firstly decreases and then increases with the increase of internal flow velocity. The maximal CF VIV amplitude with $v = 2$ appears largest among the investigated cases when KC = 100. While the internal flow velocity keeps increasing to 4, the maximal CF VIV amplitude decreases dramatically, leading to a minimum value of CF VIV amplitude compared with those with different internal flow velocities for KC = 100, as shown in Fig. 6. Meanwhile, for high KC number, the maximal VIV amplitude in CF direction shows a continuously deceasing trend with the increase of internal flow velocity. It can be observed from Fig. 6 that the maximal CF VIV amplitude is equal to approximately 0.55 while $v = 0$ for KC = 500. When the internal flow velocity is increased up to 4, the value of maximal CF VIV amplitude is reduced to 0.4.

The change of maximal CF VIV amplitude can be attributable to a determined factor, that is the variation of reduced velocity related to natural frequencies of the flexible fluid-conveying riser. As is proved by many researchers and Fig. 3, the natural frequency decreases with the increase of internal flow velocity [47,50]. As a result, the reduced velocity related to natural frequency is changed, thereby leading to the increase or decrease of CF VIV amplitude. Another potential factor for

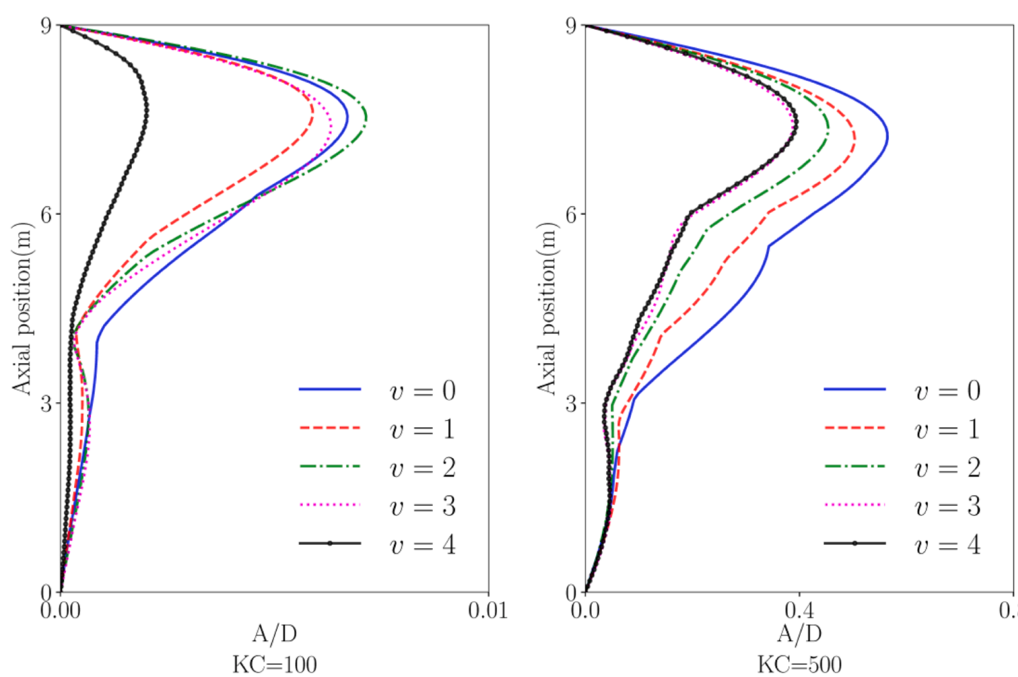

Fig. 6. Variation of maximal CF VIV amplitude with the increase of internal flow velocity under KC = 100 and 500. (The nondimensional internal flow velocity increases from 0 to 4).

the decrease of CF VIV amplitude is that the internal flow can act as a damping effect on CF VIV dynamics. When the internal flow velocity is increased, the damping effect induced by internal flow is enhanced. Consequently, maximal VIV amplitude in CF direction is reduced here.

4.3. CF VIV vibration frequency

The consideration of vessel motion and internal flow has an evident effect on CF VIV vibration frequency. In order to reveal this influence of both internal flow and vessel motion, the oscillation frequencies along the flexible fluid-conveying riser and vibration frequency of representative nodes with KC numbers equal to 100 and 500 are demonstrated in Figs. 7 and 8 while the internal flow velocity is increased.

4.3.1. Effect of vessel motion

It can be found that due to vessel motion, the oscillation frequency bandwidths for CF VIV are variable along the flexible riser. Additionally, it can be observed from Figs. 7 and 8 that more energy tends to concentrate at peak frequencies occurring in the upper section of the flexible fluid-conveying riser, which is also detected in the studies of Yuan et al. [35,41] and Xue et al. [64]. This phenomenon appears for both low and high KC numbers, as shown in Figs. 7 and 8. The reason for the characteristics of vibration frequency bandwidths and energy concentration can be explained by variable external flow velocities along the flexible riser induced by vessel motion. Since high external oscillatory flow velocity is generated near the vessel, complicated CF VIV response can be excited in the upper section of the flexible fluid-conveying riser. Whereas the external oscillatory flow velocity near the riser bottom is small, causing that no VIV response is triggered in CF direction for the corresponding section of the flexible riser. And such section is identified as the damping region. Therefore, less energy concentrates in the section near riser bottom.

4.3.2. Combination effect of vessel motion and internal flow

As the internal flow velocity is increased, the energy allocated along the flexible fluid-conveying riser is not changed conspicuously. That is, more energy still concentrates on the upper section of the flexible fluid-conveying riser. In addition, with the increase of internal flow velocity, more energy tends to concentrate at frequency peaks accordant to high mode CF VIV response while the energy concentrating on the frequency of low mode response is reduced. For instance, when KC = 500, the frequency peak corresponding to 1st mode response is obviously weakened while the 2nd and 3rd frequency peaks are strengthened with the internal flow velocity increasing, as shown in Fig. 8. Moreover, new excited CF VIV frequency can be detected when the internal flow velocity is high, especially for high KC number. In Fig. 8, the new frequency for KC = 500 corresponding to 4th mode CF VIV response can be evidently observed when v is increased up to 2 and 4. As is explained by many other researchers, the reduction of natural frequencies with the increase of internal flow velocity accounts for the appearance of new mode VIV response in CF direction, which can be also found in the works of Meng et al. [50] and Duan et al. [54,57]. Moreover, the oscillation frequency bandwidth is broadened accompanied with energy transfer. It can be found that the vibration frequency bandwidth is broader with high internal flow velocity compared to those with low internal flow velocity regardless of low and high KC numbers (Figs. 7 and 8). As for the variation of vibration frequency bandwidth, it can be ascribed to that the increase of internal flow velocity changes the energy allocation for CF VIV response via influencing CF VIV. As a result, the energy allocation on different mode VIV responses is changed, which is reflected in the form of oscillation frequency bandwidth. And the energy allocation variation occurs more conspicuously on the upper section of the flexible fluid-conveying riser due to vessel motion.

It should be emphasized that new mode CF VIV response excited with the increase of internal flow velocity is mainly captured for the flexible riser with high KC number. In the case of KC = 100, the appearance of new frequency can scarcely be detected with the increase of internal flow velocity. However, there indeed exist two vibration frequencies during our simulation, as shown in Fig. 7. This is because the existence of more than one vibration frequency for KC = 100 is related to top tension

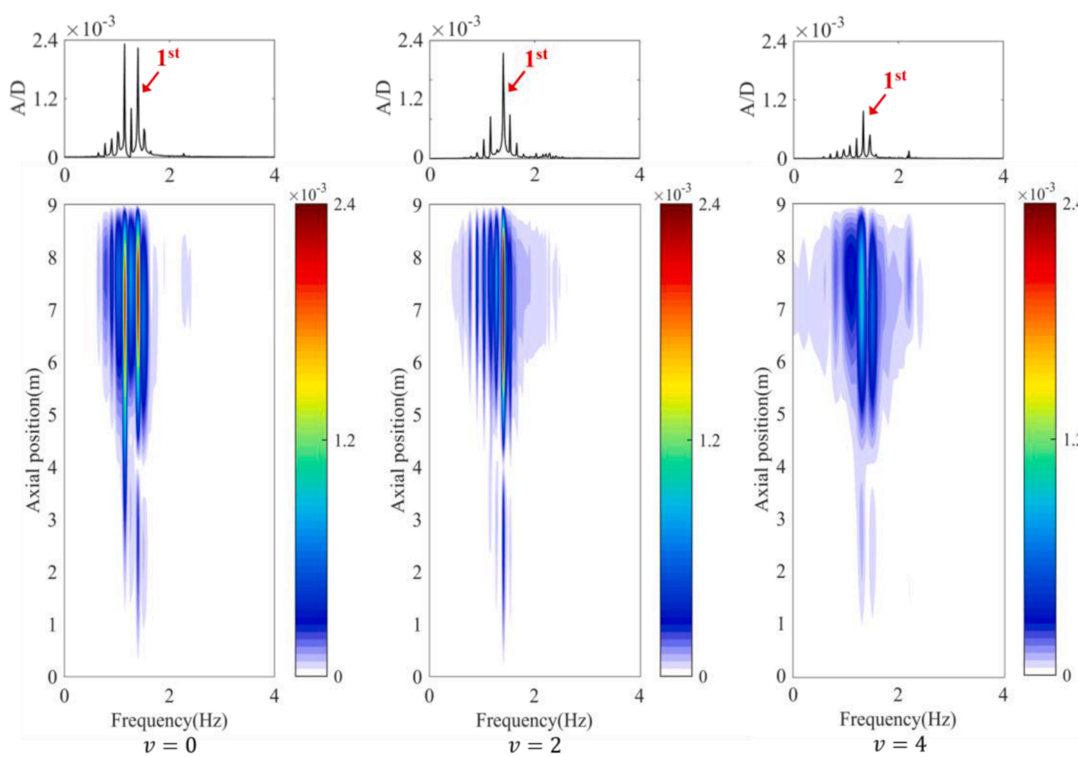

Fig. 7. Variation of CF VIV vibration frequency with the increase of internal flow velocity under KC = 100. (Above: representative vibration frequency of node 75 corresponding to the excited mode; Bottom: amplitude spectra along the flexible riser).

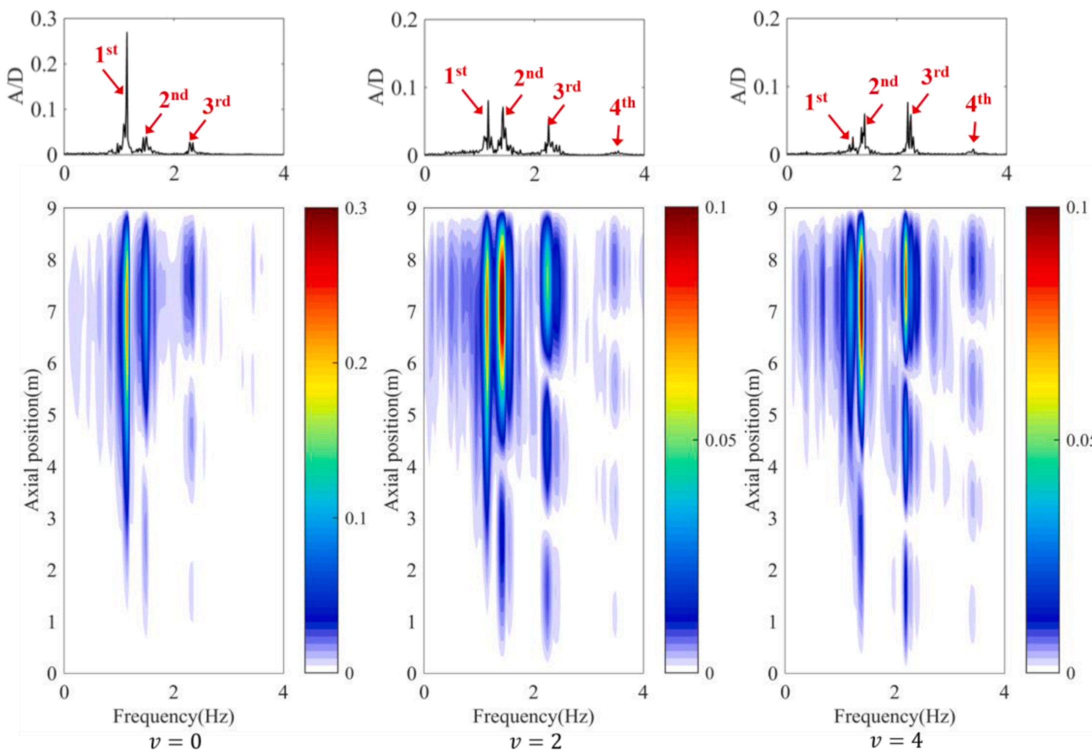

Fig. 8. Variation of CF VIV vibration frequency with the increase of internal flow velocity under KC = 500. (Above: representative vibration frequency of node 75 corresponding to the excited mode; Bottom: amplitude spectra along the flexible riser).

variation. While the internal flow velocity is increased, the effect of variable top tension is mitigated. More detailed interpretation about this phenomenon will be elaborated in the following subsection.

4.4. Time dependent CF VIV response and frequency distribution

When vessel motion is considered, the external flow velocity changes periodically. As a result, intermittent VIV occurs, leading to time-dependent VIV features. Hence, it is necessary to analyze and discuss these time-dependent VIV characteristics while both vessel motion and internal flow are taken into account. Correspondingly, the time history and vibration frequency distribution of CF VIV response with different non-dimensional internal flow velocities under low and high KC numbers are depicted in Figs. 9 and 10. Representative results of node 75 are chosen to be presented, analyzed and discussed. And continuous wavelet-transform technique is utilized to analyze oscillation frequency varying with time.

4.4.1. Effect of vessel motion

For high KC number, it can be seen that multimode and intermittent VIV, amplitude modulation, mode jump and frequency transition can be reproduced during our simulation, which can also be observed in the works of Fu et al [29], Yuan et al [35,41] and Xue et al [64]. While CF VIV characteristics, such as intermittent VIV and amplitude modulation, are mainly captured for low KC number. Obviously, intermittent VIV can be easily detected regardless of the internal flow velocity, as shown in Figs. 9 and 10. It can be found that when the external flow velocity is small for KC = 500, low mode response dominates CF VIV. As the external flow velocity is increased due to vessel motion, the corresponding high mode VIV is excited in CF direction. For example, for KC = 500, the 1st and 2nd vibration frequencies are mainly detected for CF VIV at small external flow velocity when $v = 0$, indicating that the first two mode VIV responses are excited under such circumstance (Fig. 10). When the external flow velocity is up to a certain value, the oscillation frequency corresponding to the 3rd mode CF VIV response is captured, which means that new mode VIV response in CF direction is triggered, as shown in Fig. 10. Then the external flow velocity begins to decrease after reaching its maximum due to vessel motion. Consequently, 3rd mode VIV in CF direction starts to decay and eventually disappears. At the same time, 1st and 2nd mode CF VIV responses are excited again, as observed in Fig. 10. The reason for this phenomenon is that since the external flow velocity is changed periodically with vessel motion, various mode CF VIV responses can be triggered according to the variable external flow velocity. Therefore, there exists intermittent VIV in CF direction due to vessel motion.

Since frequency transition occurs concomitant with mode jump due to variable external flow velocity, it is easy to capture frequency transition for CF VIV with high KC number. It can be seen from Fig. 10 that when the external flow velocity is increased due to vessel motion under KC = 500, CF VIV frequency transitions from 1st and 2nd frequencies 1.14Hz and 1.5Hz to the 3rd frequency 2.28Hz. As the external flow velocity is reduced, the 3rd frequency disappears gradually while the 1st and 2nd frequencies appear again (Fig. 10). Hence, this alternative occurrence of oscillation frequency, known as frequency transition, appears with variable external flow velocity due to vessel motion. It should be mentioned that the new frequency appears accompanied with the decay of existing frequency for CF VIV, which is defined as hysteresis by Fu et al. [29]. And because of the effect of hysteresis, the frequencies corresponding to low mode CF VIV response almost exist with time varying, as seen in Fig. 10.

It should be noticed that though intermittent CF VIV under low KC number also exists, it is not conspicuous compared to that with high KC number. It can be seen from Fig. 9 that for KC = 100, the CF VIV amplitude is changed with variable external flow velocity due to vessel motion. Noteworthy is that vessel motion has an obvious effect on the vibration frequency of CF VIV, especially for low KC number. During our simulation, low KC number equal to 100 is chosen. Based on KC = 100, it can be calculated that only the 1st frequency corresponding to the first mode CF VIV response is theoretically supposed to be detected. However, there exist two oscillation frequencies 1.2Hz and 1.4Hz for KC =

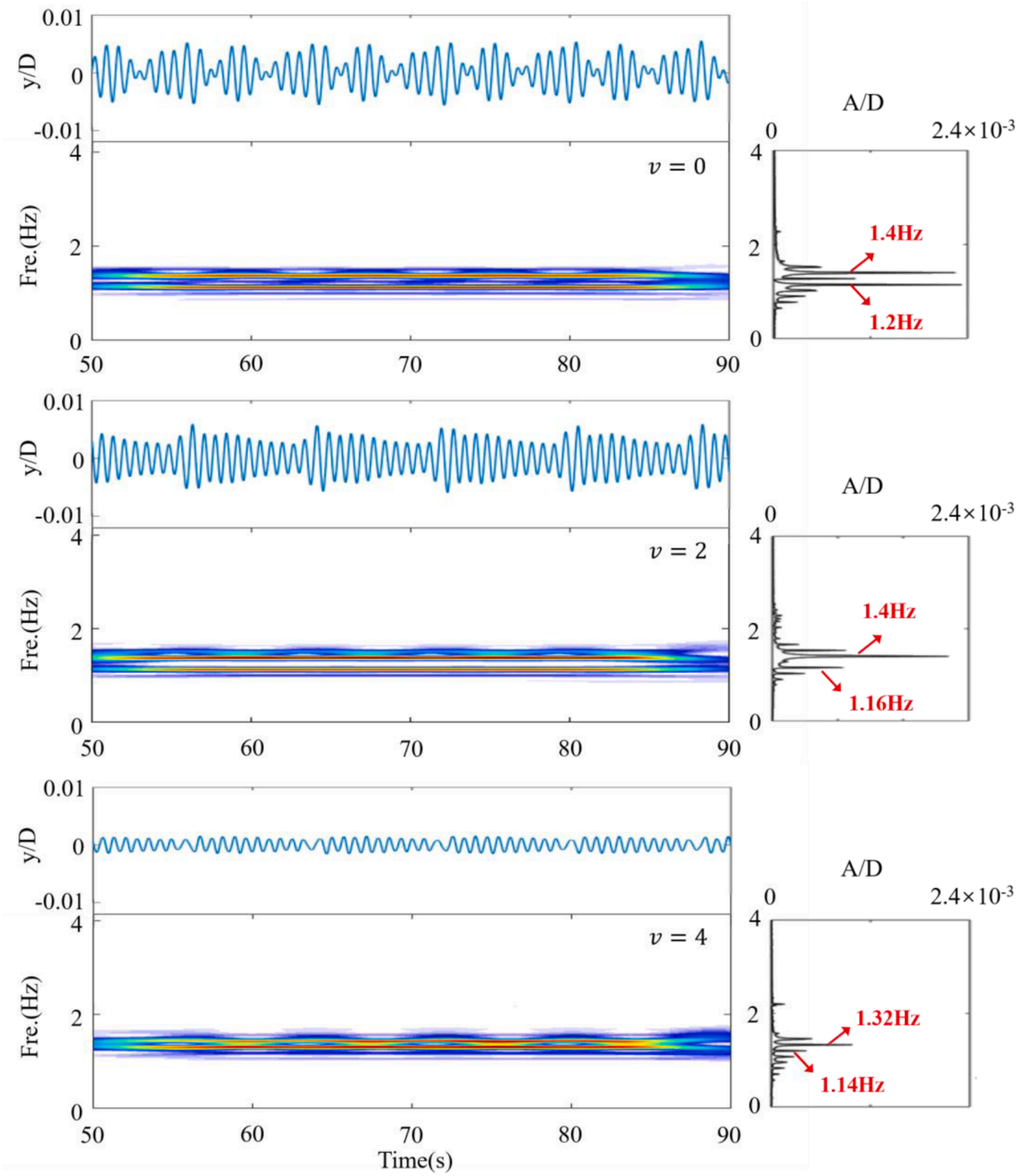

Fig. 9. Variation of CF VIV response and frequency distribution with time varying while the internal flow velocity is increased under KC = 100. (Left: time history and frequency distribution; Right: the corresponding oscillation frequency).

100 under $v = 0$. Through these two detected vibration frequencies, it can be deduced that this frequency other than the one of CF VIV is related to the frequency of vessel motion. A relationship expressed as $f' = f_v + 2f_m$ can be found between these two detected frequencies of CF VIV with KC = 100, where f_v and f_m denote the CF VIV response frequency and the frequency of vessel motion respectively. And the relationship found here is similar to that stated by Yuan et al. [26]. The existence of the frequency other than CF VIV frequency can be attributable to the influence of vessel motion on CF VIV response of the flexible fluid-conveying riser. When the riser top moves with the vessel on the surface, variable external flow current with a low frequency is induced. Correspondingly, VIV response in CF direction is influenced through the change of external hydrodynamic force due to variable external flow current, causing the appearance of an oscillation frequency related to vessel motion. The results prove that CF VIV response of the flexible fluid-conveying riser can be evidently affected by vessel motion with low KC number. It should also be noticed that the influence of vessel motion on CF VIV under high KC number is not obvious, which can be observed in Fig. 10.

Another notable feature for CF VIV due to vessel motion is that the amplitude modulates with the change of external flow velocity. As shown in Figs. 9 and 10, the minimum CF VIV amplitude appears while the external flow velocity is small for both low and high KC numbers. With the increase of external flow velocity, the CF VIV amplitude is increased. When the external flow velocity decreases, the value of CF VIV amplitude is reduced again. The amplitude modulation for CF VIV due to vessel motion can be ascribed to the change of both external hydrodynamic force and reduced velocity caused by variation of external flow velocity.

4.4.2. *Combination effect of vessel motion and internal flow*

When the flexible riser transports fluid inside, the amplitude and frequency for CF VIV are evidently influenced. It is also observed that with the increase of internal flow velocity, maximal CF VIV amplitude shows a decreasing trend. As shown in Figs. 9 and 10, the maximal CF VIV amplitude decreases with the increase of internal flow velocity, particularly for high KC number. Besides, new frequency corresponding to high mode VIV response occurs under KC = 500 while internal flow

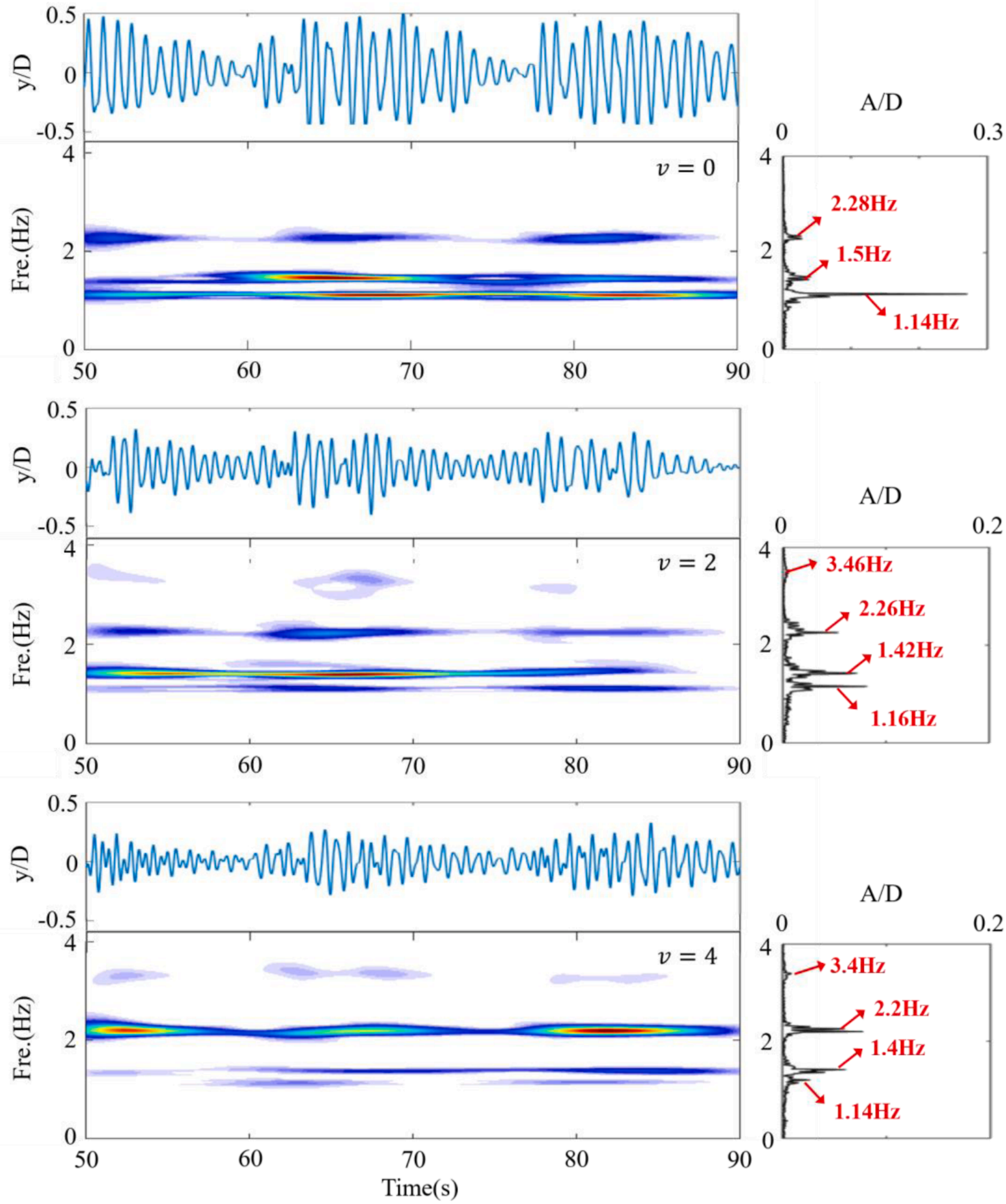

Fig. 10. Variation of CF VIV response and frequency distribution with time varying while the internal flow velocity is increased under KC = 500. (Left: time history and frequency distribution; Right: the corresponding oscillation frequency).

velocity is increased. For example, when $v = 0$, three oscillation frequencies 1.14Hz, 1.5Hz and 2.28Hz close to the first three natural frequencies 1.14Hz, 1.45Hz and 2.30Hz are detected for CF VIV under KC = 500 (Figs. 3 and 10). In terms of CF VIV vibration frequencies with $v = 2$, the values show a decreasing trend compared with those with $v = 0$. It should be noted that new vibration frequency 3.46Hz close to the 4th natural frequency of the riser 3.45Hz is captured under $v = 2$, indicating that new mode CF VIV response is triggered. As nondimensional internal flow velocity is increased up to 4, the oscillation frequencies for CF VIV keep decreasing to 1.14Hz, 1.4Hz, 2.2Hz and 3.4Hz., which corresponds to the values of natural frequencies 1.10Hz, 1.36Hz, 2.21Hz and 3.38Hz respectively (Figs. 3 and 10). The decrease of CF VIV oscillation frequencies can be attributed to the reduction of natural frequencies due to the increase of internal flow velocity, as shown in Fig. 3.

In addition, energy concentrating at low mode VIV response reduces while more energy aggregates on frequency of high mode CF VIV with the internal flow velocity increasing. As shown in Fig. 10, energy mainly concentrates on the frequency peak corresponding to the 1st mode CF VIV response when $v = 0$. With the increase of internal flow velocity, energy concentration on the 2nd and 3rd vibration frequency peaks shows an increasing trend. As v keeps increasing up to 4, more energy tends to aggregate on the frequency peak accordant to 3rd mode response while the energy allocated at 1st and 2nd mode responses is decreased. As is explained in Section 4.2, these phenomena are attributable to the change of reduced velocity and damping effect caused by internal flow increasing. It should be mentioned that for low KC number, the increase of internal flow velocity scarcely mitigates the effect of vessel motion on vibration frequency of CF VIV. As shown in Fig. 9, there always appear two oscillation frequencies for CF VIV while the internal flow velocity is increased, which indicates that vessel motion can notably affect CF VIV with low KC number.

5. Conclusions

VIV response in CF direction of a flexible fluid-conveying riser is investigated while vessel motion is taken into account during our simulation. Firstly, the governing equation of the flexible fluid-conveying riser and the applied external hydrodynamic force model are briefly introduced. Based on validation between numerical and experimental results, the feasibility and accuracy of the proposed model are examined. Then CF VIV response of a flexible riser is explored while both vessel motion and internal flow velocity are considered. And the variation of top tension and CF VIV characteristics under low and high KC numbers due to vessel motion is studied with the increase of nondimensional internal flow velocity.

It is found that top tension and CF VIV characteristics of a flexible fluid-conveying riser with both low and high KC numbers can be affected by both vessel motion and internal flow, with a dominated impact induced by vessel motion. For instance, frequencies of variable top tension related to both vessel motion and CF VIV are detected respectively. Moreover, there exists one frequency twice as that of vessel motion. And frequency of top tension twice as those of CF VIV can also be captured, especially for high KC number. The maximal top tension reduces with the increase of internal flow velocity.

As for CF VIV amplitude, the appearance of its maximal values is captured on the upper section of the flexible fluid-conveying riser for both low and high KC numbers. The maximal VIV amplitude in CF direction tends to decrease while the internal flow velocity is increased. The minimum value of CF VIV amplitude can be observed with high internal flow velocity.

Due to vessel motion, the vibration frequency bandwidth for CF VIV is changed along the flexible riser, with more energy concentration at the upper section of the flexible fluid-conveying riser. With the increase of internal flow velocity, the oscillation frequency bandwidth is broader regardless of low and high KC numbers. Besides, new frequency can be detected for CF VIV under high KC number while internal flow velocity is increased.

Intermittent CF VIV, amplitude modulation, mode jump as well as frequency transition can be reproduced for CF VIV of a flexible fluid-conveying riser under low and high KC numbers while vessel motion is taken into account. The increase of internal flow velocity can exert an obvious impact on mode jump and frequency transition. While internal flow velocity is increased, the vibration frequencies of CF VIV decrease. In addition, oscillation frequencies of CF VIV can be affected by the variable top tension, especially under low KC number.

In present work, CF VIV response of a flexible riser is studied with the consideration of vessel motion and internal flow. Correspondingly, variation of top tension and CF VIV characteristics with different KC numbers and internal flow velocities is analyzed and discussed, which could help us to acquire basic knowledge of CF VIV mechanism and features in real ocean engineering. Nevertheless, more investigations considering real environmental conditions for VIV still need to be conducted in order to understand VIV thoroughly and comprehensively.

CRediT authorship contribution statement

Jinlong Duan: Conceptualization, Methodology, Investigation, Software, Validation, Data curation, Visualization, Writing – original draft, Writing – review & editing. **Jifu Zhou:** Conceptualization, Methodology, Funding acquisition, Supervision, Visualization, Writing – review & editing. **Xu Wang:** Funding acquisition, Writing – review & editing. **Yunxiang You:** Conceptualization, Methodology.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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