## A Slip-Line Field Solution for the Ultimate Bearing Capacity of a Pipeline Under Oblique Loading on a Clayey Seabed

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Predicting the ultimate bearing capacity of a deep-sea pipeline on the soft clayey seabed is crucial to on-bottom stability and global buckling analyses. Based on the slip-line field theory, an analytical solution is derived for the undrained bearing capacity of an obliquely loaded pipeline on a clayey seabed obeying the Tresca criterion. For a wished-in-place pipeline, the failure mechanisms and the corresponding pipe-soil interface conditions are proposed for the vertical and the lateral instability mode separately. Based on the derived slip-line field solution, a series of failure envelopes are established for various pipe-soil interface conditions. The analytical solutions match well with the previous numerical and theoretical results. The parametric study indicates that the embedment-to-diameter ratio and the loading angle are the key parameters for the bearing capacity of an obliquely loaded pipeline.

## **NOMENCLATURE**

- D Outer diameter of the pipe
- e Embedment of the pipe
- f Shear stress at the pipe-soil interface
- F Bearing capacity of the slip-line field solution
- $F_{\rm I}$  Bearing capacity for mode I
- $F_{\rm II}$  Bearing capacity for mode II
- $F_{\rm x}$  Horizontal component of the soil resistance
- $F_{\rm v}$  Vertical component of the soil resistance
- H Horizontal component of the ultimate bearing capacity
- N Normal stress on the pipe-soil interface
- $N_{\rm v}$  Coefficient of horizontal soil resistance
- $N_{\rm v}$  Coefficient of vertical soil resistance
- $q_1$  Overall uniform pressure on the slip-line field
- $q_2$  Total soil pressure considering the separation of pipe-soil interface
- R Outer radius of the pipe
- $s_{\rm u}$  Undrained strength of the soil
- $T_1$ ,  $T_2$  Transitional vectors for slip-line field solution
  - V Vertical component of the ultimate bearing capacity
  - $\alpha$  Pipe-soil interfacial friction ratio
  - $\delta$  Double of the angle from the pipe-soil interface to the first principal stress plane
  - $\varepsilon_{\rm F}$  Angle of the load direction
  - $\varepsilon_{\rm f}$  Angle of the zero-shear stress point
  - $\varepsilon_{\rm u}$  Movement angle
  - $\varepsilon_{\rm s}$  Angle of the point where the interface separates
  - $\gamma'$  Buoyant unit weight of the soil
  - $\theta$  Position angle of the integral point
  - $\theta_0$  Half of the embedment angle
- $\theta_1 \sim \theta_4$  Limit of integration along the pipe-soil interface
  - $\sigma_0$  Normal stress of the stress circle's center
  - $\psi$  Angle from x axis to the first principal stress plane

## INTRODUCTION

Soft clayey sediments are widely distributed in the deep waters (see Liu et al., 2021). The deep-sea pipelines are commonly laid directly on the seabed, where the pipelines may penetrate partially or even completely into the soil during their service period (DNV GL, 2017). Under the action of ocean currents, a partially embedded pipeline would suffer an oblique hydrodynamic loading, including the drag (horizontal) and the lift (vertical) force. While a deep-sea pipeline transports oil and gas with high temperature and high pressure, axial compressive forces could be induced along the pipeline (Taylor and Gan, 1986). Under the external (hydrodynamic forces) and/or the internal (axial compressive forces due to the rise of temperature and pressure) loads, the pipeline may break away from its original location while forfeiting the on-bottom stability (Gao et al., 2007), and even buckle globally along the seabed surface (Bruton et al., 2006). Therefore, a reliable assessment of the ultimate bearing capacity for an obliquely loaded pipeline on the clayey seabed is essential to predicting the instability of deep-sea pipelines (Fredsøe, 2016; Gao, 2017).

To predict the vertical bearing capacity of submarine pipelines, empirical or semi-empirical solutions have been proposed by simplifying the circular pipe-soil interface as a rectangular (Small et al., 1971) or a wedged foundation (Karal, 1977). Based on the limit analysis of plasticity theory, an upper bound solution for the pipe-soil interaction was derived by Randolph and White (2008). The bearing capacity of the pipeline can be consequently solved using the upper bound solution. Nevertheless, the solution is implicit and mainly applicable to partially embedded pipelines. Based on the slip-line field theory, Gao et al. (2013) derived an explicit analytical solution for the vertical bearing capacity of a pipeline partially embedded or fully buried in a clayey seabed. To predict the lateral instability of a pipeline in ocean currents, a pipe-soil interaction model was established by Gao et al. (2016) using the limit equilibrium approach, in which Coulomb's theory of passive earth pressure for the sloping seabed was incorporated.

Numerical models have been intensively employed for simulating pipe-soil interactions. By applying the arbitrary Lagrangian-Eulerian (ALE) remeshing technique, Merifield et al. (2009) simulated the vertical penetration process of a pipeline. A pipe-soil interaction model was established by Tian and Cassidy (2011), which can predict large lateral displacements of the pipe in the

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Received June 28, 2022; updated and further revised manuscript received by the editors September 28, 2022. The original version (prior to the final updated and revised manuscript) was presented at the Thirty-second International Ocean and Polar Engineering Conference (ISOPE-2022), Shanghai, China (virtual), June 5–10, 2022.

KEY WORDS: Bearing capacity, deep-sea pipeline, oblique loading, slipline field theory, failure envelope, pipe-soil interaction, clayey seabed.