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Letter

The effect of gravity on self-similarity of Worthington jet after water entry of a two-dimensional wedge

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ABSTRACT

The effect of gravity on the self-similarity of jet shape at late stage of Worthington jet development is investigated by experiment in the study. In addition, the particle image velocimetry (PIV) method is introduced to analyze the development of flow field. There is a linear scaling regarding the axial velocity of the jet and the scaling coefficient increases with the Froude number.

When an object impacts a free surface, an open cavity is usually produced. After the cavity pinch-off, a liquid column called Worthington jet forms as shown in Fig. 1. The jet moves rapidly upward resulting in the violent deformation of the water surface which occurs widely in nature and engineering application and has increased attentions.

The investigations of Worthington jet have been carried on for nearly a century since Worthington [1] recorded the production of jet after water entry using single-spark photography for the first time. The jet also exists in the fields such as drop impact [2–8] and bubble bursting [9–11]. There are two main concerns in the studies of Worthington jet, i.e., the formation mechanism and dynamic characteristics of the jets. Gekle et al. established theoretical models to describe the causes of jet formation [12] and to predict the velocity and shape of jet [13]. Jamali et al. [14] studied the influences of liquid viscosity on the shape, velocity and maximum height of the jet. Guleria et al. [15] investigated the jet development induced by water entry of the sphere with different sizes and impact velocities and found that the wettability of the sphere surface is also an important factor affecting the pattern of the open cavity and subsequent generation of the jet by changing the contact lines among the phases of air, liquid, and air. McKown [16] studied the effects of hydrophilic and hydrophobic surface on formation mechanism of the jet via experiments. Fan et al. [17] argued that the pinch-off model of the open cavity directly affects the formation of the jet. The formation and development of the jet induced from the interfaces between different liquids have also been studied [18–21]. In addition, the geometry of subject entering water also brings differences to the dynamics of Worthington jet [22,23].

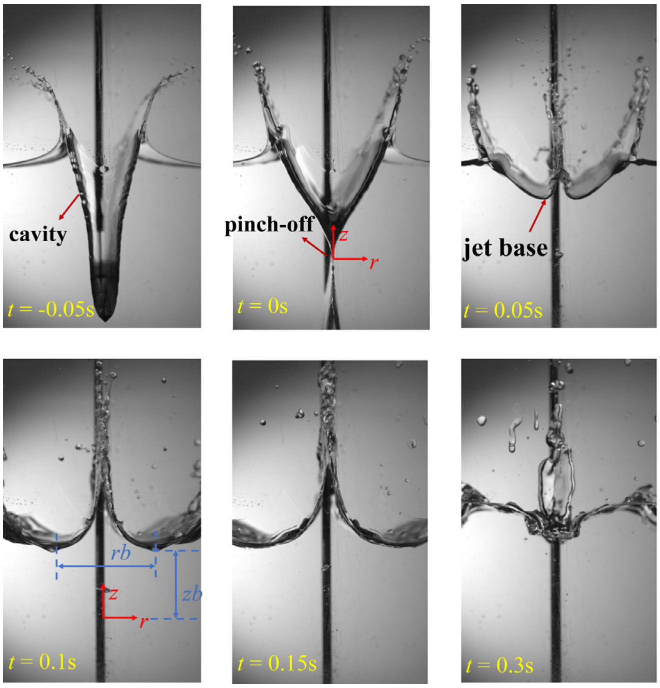

Fig. 1. The development of an open cavity and the generation of Worthington jet after the impact of a two-dimensional wedge. The initial time is set to the time when the pinch-off happens. Negative time means the time before pinch-off.

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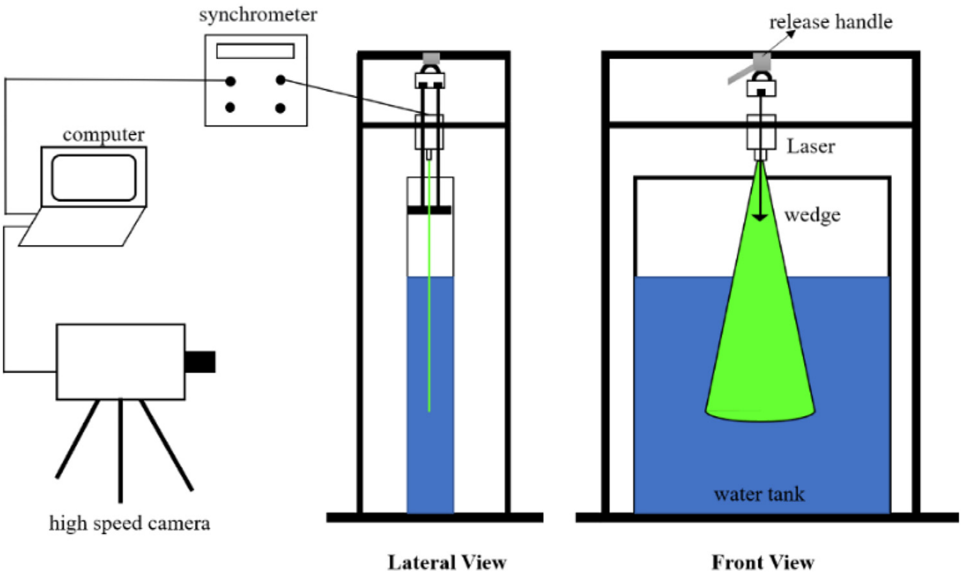

Fig. 2. The experiment setup for measuring the velocity field of the jet via PIV method after the impact of a two-dimensional wedge.

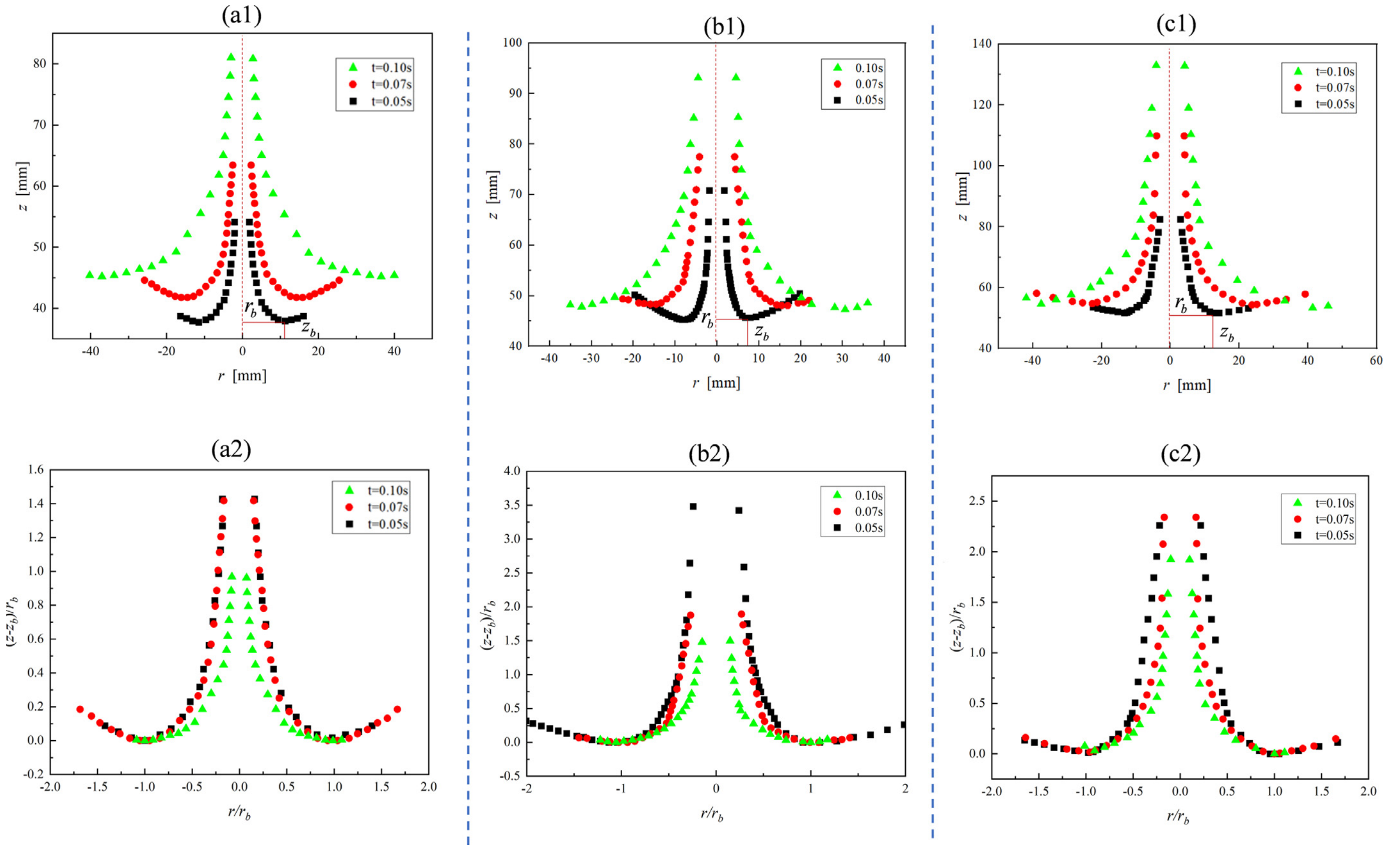

Fig. 3. The jet shape evolution with time based on high speed photography. (a1) and (a2): Fr=15; (b1) and (b2): Fr=40; (c1) and (c2): Fr=65. The first row: the initial shape of the jet over time. The examples of height and radius of jet base z_b and r_b are shown. Noting that the jet base varies with time. The second row: the corresponding jet shape nondimensionalized by the radius of jet base r_b. Here Fr is abbreviation of Froude number, Fr $= U_0^2/gL$where U_0 is the velocity of water entry and L is the length of top edge of the wedge, g is the gravitational acceleration.

Self-similarity is an important characteristic for Worthington jet. Gekle et al. [13] found that the shape of Worthington jet shows self-similarity to some degree when normalized by the radius of jet base (the lowest position of the jet, see Fig. 1). Van Rijn et al. [24] experimentally analyze the fluid flow inside the jet using particle image velocimetry (PIV) and found that the velocity profile is self-similar when the effect of gravity can be ignored. Although the mentioned-above studies about self-similarity ignore the effect of gravity, the gravity takes responsible the behaviors of the jet in the later stage of the development, i.e., when the jet starts to decelerate. Consequently, it requires to obtain detailed information on the shape and velocity of the jet. It is challenging to measure the flow field via PIV method because of complicated factors

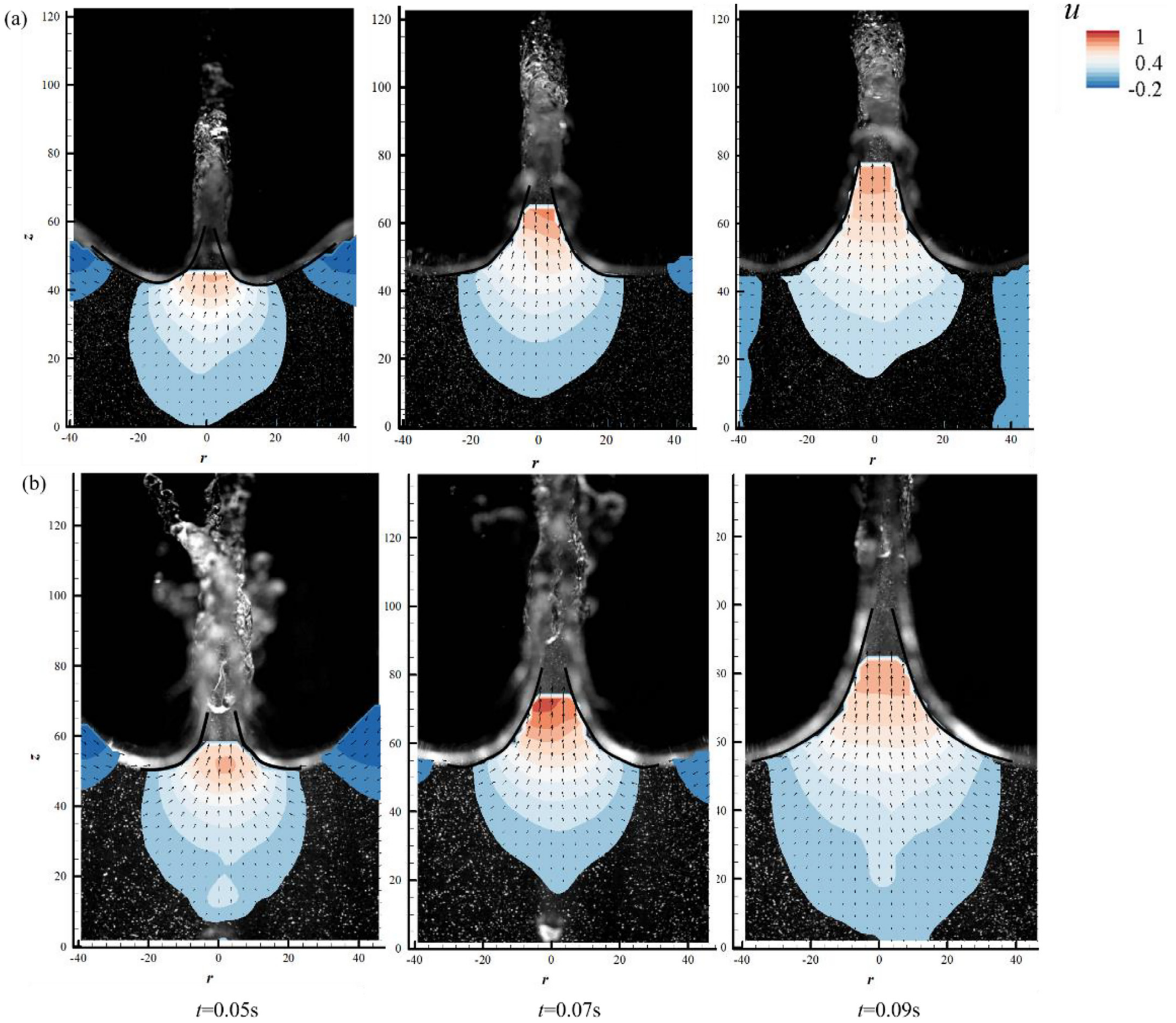

Fig. 4. The velocity fields of the jet obtained with PIV method: (a) Fr=40 (b) Fr=65. The amplitudes of the axial velocity u are presented by the color bars, while the direction is indicated by the arrows.

such as the distortion caused by three-dimensional gas-liquid interface, the strong reflection of laser at interface [25] and the narrow shape of jet. Wang et al. [26] conducted an experiment of quasi-two-dimensional cavity to prevent the effect of three-dimensional interface on particle image which we follow in this paper.

In the present study, we carry out an experiment of water entry of two-dimensional wedge and introduce the PIV method to analyze the development of the Worthington jet. The results show that the gravity will make the self-similarity of jet shape disappear in later stage of jet development. In addition, the scaling law of jet velocity including the contribution of the gravity is obtained. The study can provide a new insight to analyze the Worthington jet.

The experiment setup for measuring the velocity jet via PIV method is shown as Fig. 2. The system of water entry launch is conducted based on the study by Wang et al. [29]. In the experiments, a two dimensional wedge with top edge of 10 mm, deadrise angle of 45° and the spanwise length of 150 mm is used. The size of water tank is 150 mm in width and 600 mm in length and 800 mm in height. The cavity interface created by water entry of the wedge is almost two-dimensional since its spanwise is the same as the width of water tank. Therefore, the problem that the particles images distorted by 3D interface will not happen. A 2D planar PIV measurements are conducted in the water tank with a pulsed laser (527 nm, 40 mJ, Beamtech Optronics Co., Ltd.). Fluorescence particles with diameter of 20 μm are seeded in water. The corresponding Stokes number is much smaller than 1 which meets the requirements of particle selection. Flow fields and jet shape is recorded by a high-speed camera (Phantom V1612, Vision Research) with a SIGMA 105mm/f2.8 macro lens. The frame rate is set to 1000 fps with resolution of 1280×800 pixels.

The self-similarity of jet shape studied by Gekle et al. [13] is at the early stage of development when the effect of gravity can be ignored. Figure 3a1-c1 show the evolutions of jet shapes with time based on high-speed photography for Fr=15, 40 and 65 at the later stage. The abscissa r and ordinate z are the radial direction and the axial direction, respectively. After the jet forms, it widens and elongates in the radial and axial directions, respectively. The height of the jet base z_b increases firstly

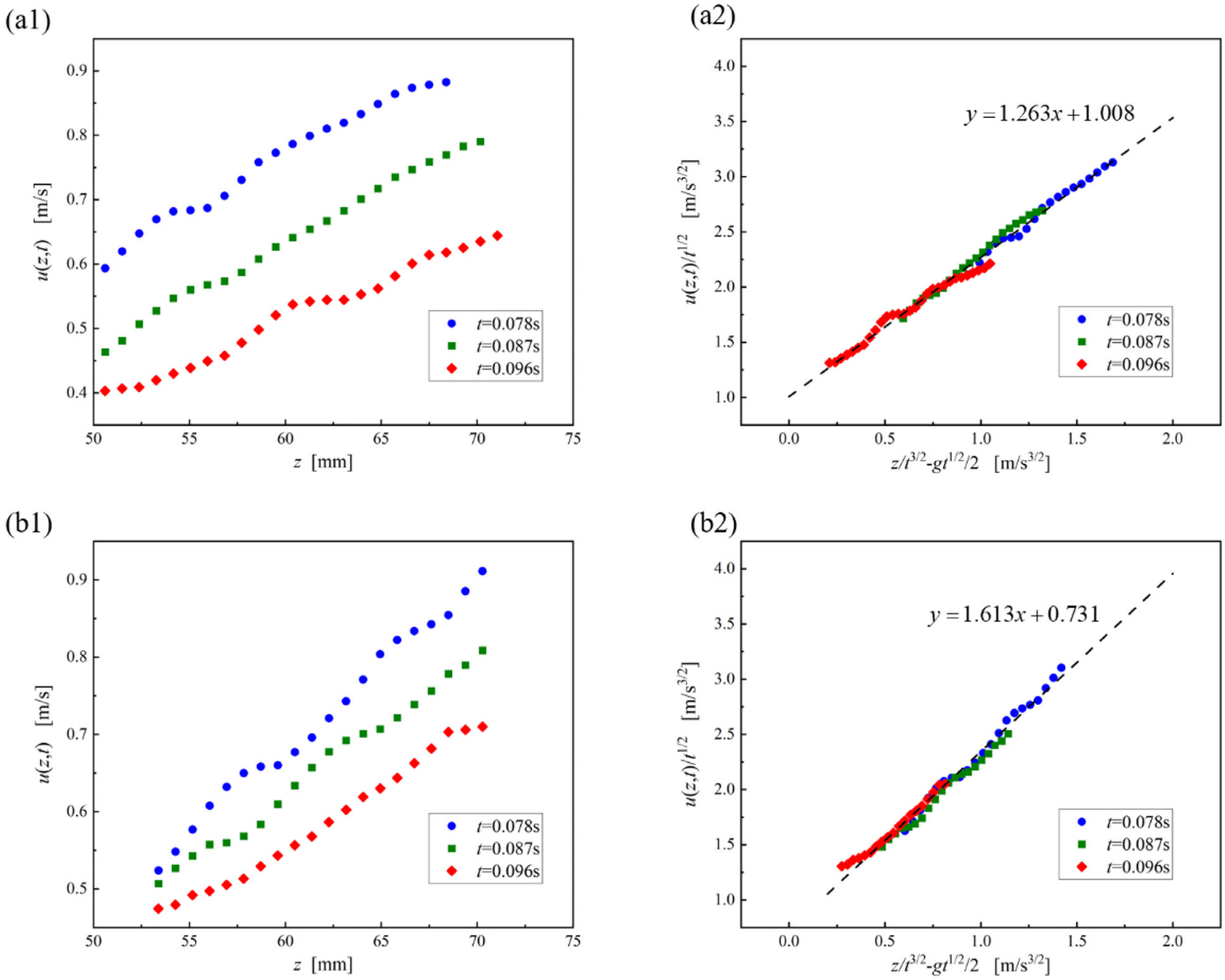

Fig. 5. Distribution of the axial velocity u in z direction with time and the corresponding rescaled velocity profile. (a1) and (a2): Fr=40; (b1) and (b2): Fr=65.

and then almost remains while the radius of jet base r_b continuously increases with the time. The jet shapes are nondimensionalized by the radius of jet base referring to the study by Gekle et al. [13], as shown in Fig. 3a2-c2. At Fr=15 and 40, the outlines at t=0.05 s and t=0.07 s are similar, but the shape at t=0.10 s shows difference. For Fr=65, the dimensionless jet shapes for the three moments deviate from each other which indicate that the similarity disappears. The time when gravity starts to affect the similarity is different, for Fr=15 and 45, is after t=0.05 s and before t=0.07 s; for Fr=65, is before t=0.05 s.

The flow fields of the jets at Fr=40 and Fr=65 are shown in Fig. 4. It can be seen that the axial velocity increases continuously from the bottom to the top for the jet. The velocity section at the bottom of the jet has larger radial velocity, and the radial velocity decreases at the upper portion, the velocity tends to be uniform in the axial direction. To further explore the evolutions of axial velocity, we extract the variation of the axial velocity u at the axis of symmetry (r=0) with the axial coordinate z as shown in Fig. 5a1 and b1. It shows that the velocity decreases with time at the same value of z.

Van Rijn et al. [24] experimentally verified a linear scaling between $z/t^{3/2}$ and $u/t^{1/2}$ when gravity contribution can be ignored and the slope of the scaling line is equal to 1. Considering the effect of gravity, we plot the variation of $u/t^{1/2}$ with $(z/t^{3/2}$- $gt^{1/2}/2)$ for different Froude number as shown in Fig. 5a2 and b2. The results show that a linear scaling between $u/t^{1/2}$ with $(z/t^{3/2}$- $gt^{1/2}/2)$ occurs at different Froude numbers. Furthermore, the slopes are higher than 1 when considering the effect of the gravity. In addition, the slope increases with the Froude Number. Based on the scaling law between $(z/t^{3/2}$- $gt^{1/2}/2)$ and $u/t^{1/2}$, one can calculate velocity at certain height for a given time. In this way, even though the velocity field of the upper part of the jet cannot be obtained by PIV because of the low quality of particle images, we can predict it by the scaling relation.

In conclusions, the effect of gravity on the self-similarity of Worthington jet is analyzed. The results of high-speed photography show that the self-similarity of jet shape disappear as the effect of the gravity dominates the development of the jet. The velocity fields of Worthington jet are obtained by the PIV method. It is found that there is a linear scaling between $u/t^{1/2}$ and $(z/t^{3/2}$- $gt^{1/2}/2)$ when considering the effect of the gravity. The scaling law provides a new thought to predict the velocity which is hard to be got by experiment.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] A.M. Worthington, R.S. Cole, Impact with a liquid surface, studied by the aid of instantaneous photography, Phil. Trans. Roy. Soc. Lond. 189 (1897) 137–148.

[2] A.U. Siddique, M. Trimble, F. Zhao, M.M. Weislogel, H. Tan, Jet ejection following drop impact on micropillared hydrophilic substrates, Phys. Rev. Fluid. 5 (6) (2020) 063606.

[3] B. Zhang, V. Sanjay, et al., Impact forces of water drops falling on superhydrophobic surfaces, Phys. Rev. Lett. 129 (10) (2022) 104501.

[4] A. Singh, P. Kumar, Droplet impact dynamics onto a deep liquid pool of wavy free surface, Phys. Fluid. 34 (2) (2022) 022107.

[5] S.T. Thoroddsen, K. Takehara, H.D. Nguyen, T.G. Etoh, Singular jets during the collapse of drop-impact craters, J. Mech. 848 (2018) R3.

[6] K. Yamamoto, M. Motosuke, S. Ogata, Initiation of the Worthington jet on the droplet impact, Appl. Phys. Lett. 112 (9) (2018) 093701.

[7] Q. Ding, T. Wang, Z. Che, Two jets during the impact of viscous droplets onto a less-viscous liquid pool, Phys. Rev. E 100 (5) (2019) 053108.

[8] D. Kim, J. Lee, A. Bose, I. Kim, J. Lee, The impact of an oil droplet on an oil layer on water, J. Fluid Mech. 906 (2020) A5.

[9] F.J. Blanco-Rodriguez, J.M. Gordillo, On the sea spray aerosol originated from bubble bursting jets, J. Fluid Mech. 886 (2020) R2.

[10] A. Zhang, S. Li, P. Cui, et al., A unified theory for bubble dynamics, Phys. Fluid. 35 (3) (2023) 033323.

[11] J.M. Gordillo, J. Rodriguez-Rodriguez, Capillary waves control the ejection of bubble bursting jets, J. Fluid Mech. 867 (2019) 556–571.

[12] S. Gekle, J.M. Gordillo, D. van der Meer, D. Lohse, High-Speed Jet Formation after Solid Object Impact, Phys. Rev. Lett. 102 (3) (2009) 034502.

[13] S. Gekle, J.M. Gordillo, Generation and breakup of Worthington jets after cavity collapse. Part 1. Jet formation, J. Fluid Mech. 663 (2010) 293–330.

[14] M. Jamali, A. Rostamijavanani, An experimental study of cavity and Worthington jet formations caused by a falling sphere into an oil film on water, Appl. Ocean Res. 102 (2020) 102319.

[15] S.D. Guleria, A. Dhar, D.V. Patil, Experimental insights on the water entry of hydrophobic sphere, Phys. Fluid. 33 (10) (2021) 102109.

[16] J.M. McKown, J. Marie, An Experimental Study of Worthington Jet Formation After Impact Of Solid Spheres, Massachusetts Institute of Technology, 2012.

[17] C. Fan, Z. Li, M. Du, Experimental study on different behaviors of spheres entering water and PEO solution, Marine Georesourc. Geotechnol. 39 (4) (2020) 471–481.

[18] D.A. Watson, J.L. Stephen, A.K. Dickerson, Impacts of free-falling spheres on a deep liquid pool with altered fluid and impactor surface conditions, J. Visual. Exper. 144 (2019) e59300.

[19] D.A. Watson, J.L. Stephen, A.K. Dickerson, Jet amplification and cavity formation induced by penetrable fabrics in hydrophilic sphere entry, Phys. Fluid. 30 (8) (2018) 082109.

[20] C.W. Tan, P.J. Thomas, Influence of an upper layer liquid on the phenomena and cavity formation associated with the entry of solid spheres into a stratified two-layer system of immiscible liquids, Phys. Fluid. 30 (6) (2018) 064104.

[21] C. Fan, X. Dong, Z. Li, Effect of upper layer immiscible liquids on the water entry phenomena, Ocean Eng. 226 (3) (2021) 108864.

[22] M.A. Jafari, P. Akbarzadeh, Experimental analysis of water entry problem considering hollow cylinders: The impact of hole geometry, Ocean Eng. 259 (2022) 111906.

[23] Y. Lee, S. Shin, G. Choi, et al., Symmetry breaking of Worthington jets by gradients in liquid pool depth, Phys. Fluid. 32 (11) (2020) 112104.

[24] C.J.M. van Rijn, J. Westerweel, B. van Brummen, et al., Self-similar jet evolution after drop impact on a liquid surface, Phys. Rev. Fluid. 6 (3) (2021) 034801.

[25] Y. Wu, Y. Liu, S. Song, J. Hong, On the internal flow of a ventilated supercavity, J. Fluid Mech. 862 (2019) 1135–1165.

[26] Y. Wang, Z. Wang, Y. Du, J. Wang, Y. Wang, C. Huang, On the airflow in a cavity during water entry, Int. J. Multiph. Flow 151 (2022) 104073.