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PAPER

Goos–Hänchen shifts on spin representation

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Abstract

We analyze the Goos–Hänchen (GH) shift and longitudinal spin splitting (LSS) at a planar interface between two optical media in the spin representation. While these optical effects have been studied previously, we examine the direct and cross-reflected light fields, and their interference from the spin representation to reveal the physical mechanism of the GH shift and establish a quantitative relationship between it and LSS. Furthermore, we show that angular asymmetric spin splitting occurs under the spin representation when linearly polarized light with a phase difference of 180° and an amplitude ratio angle deviating from 45° impinges on the air–glass interface at Brewster's angle. Finally, we reveal that the spin component field of the reflected light field for the total reflection case is different from that of the Brewster angle reflection, the most typical manifestation is that the intensity of the two spin component fields is not equal.

1. Introduction

The Goos–Hanchen (GH) shift corresponds to the longitudinal displacement of the center of the beam concerning its geometrically optically predicted, which occurs when a local wave packet (or beam) is totally reflected at the interface of two uniformly transparent media. Although the first prediction of GH displacement dates back to 1947 [1], research in this area is still very active, such as the physical mechanism of GH shift [2–7], GH shift of various beams [8–10], and GH shift on the surface of various materials [11–15]. Recently, the GH shift has not only been studied from the perspective of fundamental physics, but also photonic devices for sensing based on the GH effect have been introduced [16].

The spin Hall effect (SHE) of light is an interesting optical phenomenon, which results from the spin–orbit interactions of light. In recent years, The SHE has also been deeply studied in two kinds of single-negative metamaterials, which would pave the way for promising integrated near-field photonics devices [17, 18]. Just as the Imbert–Fedorov (IF) shift is closely related to the SHE of light [19–22], the GH shift is also closely related to longitudinal spin splitting (LSS). Back in 2011, Qin *et al* proposed a method for measuring the amplified LSS after the collimation lens using a position-sensitive detector [23]. Many interesting optical phenomena have been discovered in the study of LSS of light [24–29]. To deeply reveal the relationship between the LSS and GH shift, it is necessary to revisit the GH shift of arbitrarily polarized light in the spin representation. Fortunately, Li's pioneering work on GH displacement and IF displacement [5] shows the relationship between GH displacement and the polarization parameter of incident light in spin representation. It is important to emphasize that the concept of spin splitting does not receive attention in Li's paper, and the expression for the displacement of the two spin components of the reflected light is not given. Therefore, Li's work did not focus on the question of how GH displacement is related to LSS. Recently, a series of interesting papers have also studied the mechanism of the SHE of light from the perspective of the spin component of the incident light field [30–32], the methods presented by these works obviously cannot be used to analyze the LSS and GH shift from the perspective of spin representation of incident polarized light. This is because the mathematical relationship between the polarization parameters of the incident light field in the spin representation and the polarization parameters of the linear polarization basis representation (i.e. spin-zero representation), such as amplitude ratio angle and phase difference, is not revealed. Especially, how to define the polarization parameters of the incident light in the spin representation. In addition, how the interference phenomenon of the spin component of the light field after the reflection process subtly affects the LSS and GH shift is also not considered. The main purpose of this work is to fill these gaps.

The structure of this work is as follows: We first solve the general problem of arbitrarily polarized beams in spin representations reflecting at the planar interface between two optical media. Next, we derive the analytical expression of LSS for arbitrarily polarized light and the relation between GH shift and LSS. And then, we consider some interesting phenomena of GH shift and LSS when the incidence angle is equal to Brewster's angle. In particular, from the interference cancellation effect of the spin component of the reflected light field, we reveal the physical image of weak longitudinally symmetric spin splitting for the incident linearly polarized light with the amplitude ratio angle approaching 45° and the phase difference of 180° in the spin representation. Finally, we discuss the relationship between GH displacement and LSS in the case of total reflection and reveal that the spin component field of the reflected light field is different from that of the Brewster angle reflection, the most typical manifestation is that the intensity of the two spin component fields is not equal.

2. Theory and model

The paraxial beam propagates at an θ_i angle between the center wave vector and the z axis, and is reflected at the $z = 0$ plane separated by two media, as shown in figure 1. We assume that the incident beam is a uniformly polarized paraxial Gaussian beam with the waist at the interface $z = 0$, so that the electric field of the incident polarized light field in the laboratory coordinate system $(ox^i y^i z^i)$ can be written as [9]:

$$\left|\tilde{\boldsymbol{E}}^i\right\rangle = \begin{pmatrix} a_P^i \\ a_S^i \end{pmatrix} \tilde{\varphi}_0^i = \left[a_P^i\left|P\right\rangle + a_S^i\left|S\right\rangle\right]\tilde{\varphi}_0^i,\ \tilde{\varphi}_0^i = \exp\left(-\frac{{k_x^i}^2 + {k_y^i}^2}{4w_0^{-2}}\right) \tag{1}$$

where $k_x^i = \boldsymbol{k}^i \cdot \hat{\boldsymbol{x}}_i$, $k_y^i = \boldsymbol{k}^i \cdot \hat{\boldsymbol{y}}_i$ with $\left|\boldsymbol{k}^i\right| = \left|\boldsymbol{k}_c^i\right|$, $\left|P\right\rangle = (1,0)^\dagger_{\mathrm{LPB}}$ and $\left|S\right\rangle = (0,1)^\dagger_{\mathrm{LPB}}$, the superscript † and subscript LPB are conjugate transpose operators and two-dimensional row vectors in linear polarization basis respectively. $(a_P^i, a_S^i)^\dagger$ can be regarded as the Jones matrix under the linearly polarized representation (i.e. spin-zero representation), $a_P^i = \cos\alpha_0$, $a_S^i = \sin\alpha_0 \exp(i\Delta\phi_0)$, with α_0 denotes the amplitude ratio angle, and $\Delta\phi_0$ is the phase difference between horizontal polarization and vertical polarization components of the electric field of the incident light beam, w_0 is the waist radius of the incident beam, which reflects the collimation of the incident beam.

According to equation (1), it is not difficult to derive the electric field expression of the incident light in the spin representation as:

$$\left|\tilde{\boldsymbol{E}}_S^i\right\rangle = C_0 \begin{pmatrix} a_R^i \\ a_L^i \end{pmatrix} \tilde{\varphi}_0^i \sim \left[a_R^i\left|R\right\rangle + a_L^i\left|L\right\rangle\right]\tilde{\varphi}_0^i, \tag{2}$$

where $C_0 = (1 + \sin 2\alpha_0 \sin\Delta\phi_0)^{1/2}/[\cos\alpha_0 + i\sin\alpha_0\exp(-i\Delta\phi_0)]$ can be understood as a constant factor resulting from the normalization of polarization parameters.$\left|R\right\rangle = (1,0)^\dagger_{\mathrm{CPB}}$ and $\left|L\right\rangle = (0,1)^\dagger_{\mathrm{CPB}}$, the subscript CPB is two-dimensional row vector in circular polarization basis respectively. $(a_R^i, a_L^i)^\dagger$ can be regarded as the Jones matrix under the circularly polarized representation (i.e. the spin representation), $a_R^i = \cos\alpha_C$, $a_L^i = \sin\alpha_C \exp(i\Delta\phi_C)$, with α_C denotes the amplitude ratio angle, and $\Delta\phi_C$ is the phase difference between right-handed components and left-handed components of the electric field of the incident light beam. The relationship between the polarization parameters of the incident light beam in the circular polarization basis $(\alpha_C, \Delta\phi_C)$ and those in the linear polarization basis $(\alpha_0, \Delta\phi_0)$ can be expressed as:

$$\cos\alpha_C = \frac{(1 + \sin 2\alpha_0 \sin\Delta\phi_0)^{1/2}}{\sqrt{2}};$$

$$\sin\alpha_C = \frac{\left(\cos^2 2\alpha_0 + \sin^2 2\alpha_0 \cos^2\Delta\phi_0\right)^{1/2}}{\left[2\left(1 + \sin 2\alpha_0 \sin\Delta\phi_0\right)\right]^{1/2}} \tag{2a}$$

$$\Delta\phi_C = \tan^{-1}\left(\tan 2\alpha_0 \cos\Delta\phi_0\right). \tag{2b}$$

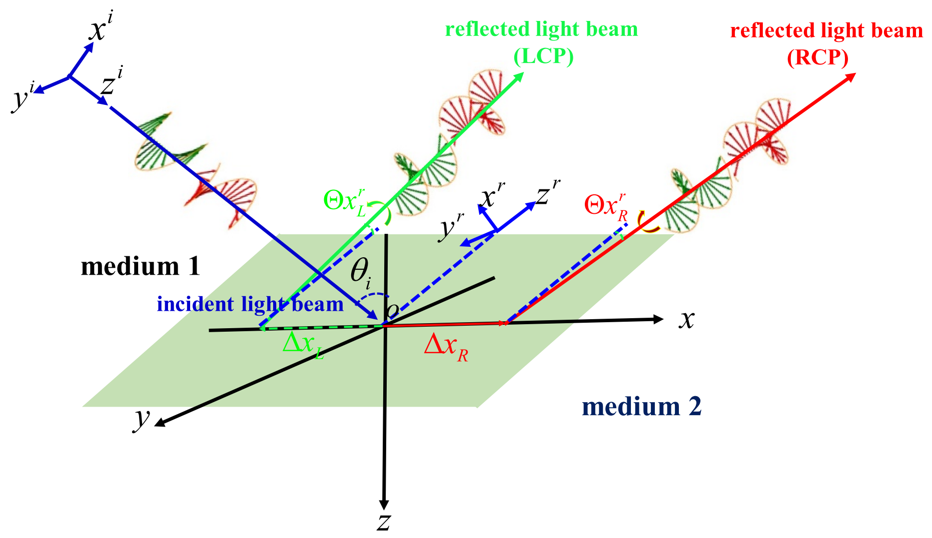

Figure 1. Schematic illustration of the LSS of an arbitrarily polarized light beam in spin representation. $\Delta x^r_{R/L}$ and $\Theta x^r_{R/L}$ represent reflected light beam undergoing an spatial and angular LSS, respectively. Where $\Delta x^r_{R/L} = \Delta x_{R/L}/\cos\theta_r$, $\theta_r{=}\arccos(\hat{\boldsymbol{z}}_r\cdot\hat{\boldsymbol{z}})$.

According to the spin representation transformation and coordinate transformation rules, the electric field of reflected light in the local laboratory coordinate system can be obtained (see appendix for detailed derivations) as

$$\left|\tilde{\boldsymbol{E}}^r_S\right\rangle = \hat{M}^r_S\left|\tilde{\boldsymbol{E}}^i_S\right\rangle,$$
$$\hat{M}^r_S = \hat{V}\hat{U}^\dagger_{\theta_r\perp}\left(\theta_r,\boldsymbol{k}_r\right)\hat{V}^\dagger\hat{S}^r\hat{F}^r_S\hat{V}\hat{U}_{\theta_i\perp}\left(\theta_i,\boldsymbol{k}_i\right)\hat{V}^\dagger. \tag{3}$$

With

$$\hat{V} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix},\hat{U}_{\theta_i\perp}\left(\theta_i,\boldsymbol{k}_i\right) = \begin{pmatrix}1 & \frac{k^i_y}{k^i}\cot\theta_i\\ -\frac{k^i_y}{k^i}\cot\theta_i & 1\end{pmatrix},$$

$$\hat{S}^r\left|\tilde{E}^r\left(k^i_x,k^i_y\right)\right\rangle = \left|\tilde{E}^r\left(-k^r_x,k^r_y\right)\right\rangle,\hat{U}_{\theta_r\perp}\left(\theta_r,\boldsymbol{k}_r\right) = \hat{U}_{\theta_i\perp}\left(\theta_i\mapsto\theta_r,\boldsymbol{k}_i\mapsto\boldsymbol{k}_r\right)$$

$$\hat{F}^r_S = \begin{pmatrix}r_{RR} & r_{RL}\\ r_{LR} & r_{LL}\end{pmatrix},r_{RR} = r_{LL} = \frac{r_P+r_S}{2};r_{LR} = r_{RL} = \frac{r_P-r_S}{2}.$$

It should be noted that Aiello and Bliokh have given detailed derivation of $\hat{V}$ matrix and $\hat{U}_{\theta_i\perp}(\theta_i,\boldsymbol{k}_i)$ matrix in their pioneering work on GH displacement and IF displacement [33–35]. By performing the inverse Fourier transform of equation (3), the coordinate representation of the electric field for the reflected light beam can be written as:

$$\left|\boldsymbol{E}^r_S\right\rangle = \left(E^r_{RR}+E^r_{RL}\right)|R\rangle+\left(E^r_{LR}+E^r_{LL}\right)|L\rangle. \tag{4}$$

With $E^r_{RR} = r_{RR}(1+\tilde{x}_{RR}\xi_{x^r}+\tilde{y}_{RR}\xi_{y^r})a^i_R\varphi^r_0$, $E^r_{RL} = r_{RL}(1+\tilde{x}_{RL}\xi_{x^r})a^i_L\varphi^r_0$, $E^r_{LR} = r_{LR}(1+\tilde{x}_{LR}\xi_{x^r})a^i_R\varphi^r_0$, $E^r_{LL} = r_{LL}(1+\tilde{x}_{LL}\xi_{x^r}+\tilde{y}_{LL}\xi_{y^r})a^i_L\varphi^r_0$.

Where $\tilde{x}_{ij} = \frac{i\partial r_{ij}}{k^r r_{ij}\partial\theta_i}(ij = RR,RL,LR,LL)$, $\tilde{y}_{RR} = -\tilde{y}_{LL} = \frac{2}{k^r}\cot\theta_i$, $\xi_{x^r} = -\frac{k^r x^r}{z_R+iz^r}$, $\xi_{y^r} = -\frac{k^r y^r}{z_R+iz^r}$, $\varphi^r_0 = \exp\left(-\frac{k^r}{2}\frac{x^{r2}+y^{r2}}{z_R+iz^i}\right)$, $z_R = k^r w_0^2/2$.

Here E^r_{RR} and E^r_{LR} are the right-handed and left-handed polarization components of the reflected light field caused by the right-handed polarization component of the incident light field respectively, while E^r_{RL} and E^r_{LL} are the right-handed and left-handed polarization components of the reflected light field caused by the left-handed polarization component of the incident light field respectively. It is not difficult to find that the right-handed polarization component of the incident light field not only contributes to the right-handed field of the reflected light field, but also contributes to the left-handed field of the reflected light field. For the convenience of the following discussion, E^r_{RR} and E^r_{LL} are called directly reflected fields, and E^r_{LR} and E^r_{RL} are called cross-reflected light fields. On the one hand, the spin shifts come from the self-intensity of the reflected spin light fields E^r_{RR} and E^r_{RL} (E^r_{LR} and E^r_{LL}), and on the other hand from the interference of the cross-reflected spin light field E^r_{RL} (E^r_{LR}) and the directly reflected spin light field E^r_{RR} (E^r_{LL}). The latter is crucial for the analysis of the GH shift generation mechanism in spin representation. For example, does it increase or decrease the intensity of the spin component of the total reflected light field, and how does it further affect the LSS behavior? Then, the expressions of the longitudinal spatial and angular spin shifts for the reflected light beam using defined spin shifts formulas $\Delta x^r_{R/L}(z^r=0)=\frac{\langle E^r_{R/L}|x^r|E^r_{R/L}\rangle}{\langle E^r_{R/L}|E^r_{R/L}\rangle}$, $\Theta x^r_{R/L}=\frac{\partial \Delta x^r_{R/L}}{\partial z^r}$ are given as:

$$\Delta x^r_R=\frac{-w_0^2\,\mathrm{Re}\left(\tilde{x}_R\right)}{w_0^2+|\tilde{x}_R|^2+|\tilde{y}_R|^2},\qquad \Delta x^r_L=\frac{-w_0^2\,\mathrm{Re}\left(\tilde{x}_L\right)}{w_0^2+|\tilde{x}_L|^2+|\tilde{y}_L|^2},\tag{5}$$

$$\Theta x^r_R=\frac{1}{z_R}\frac{-w_0^2\,\mathrm{Im}\left(\tilde{x}_R\right)}{w_0^2+|\tilde{x}_R|^2+|\tilde{y}_R|^2},\qquad \Theta x^r_L=\frac{1}{z_R}\frac{-w_0^2\,\mathrm{Im}\left(\tilde{x}_L\right)}{w_0^2+|\tilde{x}_L|^2+|\tilde{y}_L|^2}.\tag{6}$$

With

$$\tilde{x}_R=i\frac{1}{k^i}\frac{\frac{\partial r_{RR}}{\partial\theta_i}a^i_R+\frac{\partial r_{RL}}{\partial\theta_i}a^i_L}{r_{RR}a^i_R+r_{RL}a^i_L},\tilde{x}_L=i\frac{1}{k^i}\frac{\frac{\partial r_{LR}}{\partial\theta_i}a^i_R+\frac{\partial r_{LL}}{\partial\theta_i}a^i_L}{r_{LR}a^i_R+r_{LL}a^i_L},\tilde{y}_R=\frac{1}{k^i}\frac{2r_{RR}a^i_R}{r_{RR}a^i_R+r_{RL}a^i_L}\cot\theta_i,\tilde{y}_L=\frac{1}{k^i}\frac{-2r_{LL}a^i_L}{r_{LR}a^i_R+r_{LL}a^i_L}\cot\theta_i.$$

According to equations (5) and (6), the spatial and angular GH displacement expressions in the spin representation can be expressed as

$$\Delta^r_{\mathrm{GH}}=\frac{\left|r_{RR}a^i_R+r_{RL}a^i_L\right|^2\Delta x^r_R+\left|r_{LR}a^i_R+r_{LL}a^i_L\right|^2\Delta x^r_L}{\left|r_{RR}a^i_R+r_{RL}a^i_L\right|^2+\left|r_{LR}a^i_R+r_{LL}a^i_L\right|^2},\tag{7}$$

$$\Theta^r_{\mathrm{GH}}=\frac{\left|r_{RR}a^i_R+r_{RL}a^i_L\right|^2\Theta x^r_R+\left|r_{LR}a^i_R+r_{LL}a^i_L\right|^2\Theta x^r_L}{\left|r_{RR}a^i_R+r_{RL}a^i_L\right|^2+\left|r_{LR}a^i_R+r_{LL}a^i_L\right|^2}.\tag{8}$$

Equations (7) and (8) describe the quantitative relationship between GH shift and LSS, and can clearly reflect the influence of the spin component of the arbitrarily polarized incident light on the LSS of the reflected light. This is why it is interesting to analyze GH shifts in spin representations. It is worth noting that equations (7) and (8) are valid for partial or total reflection of two transparent media, such as air–glass interface, and for reflection of transparent and absorbent media interface, such as an air–metal interface.

3. Brewster's angle incidence and total reflection

In this section, we focus on the relationship between GH shift and LSS in spin representation. When the $\alpha_C=\alpha_0-45°$, $\Delta\phi_C=180°$ and $\alpha_C=45°$, $\Delta\phi_C=2\alpha_0$, which correspond to elliptically polarized light with $\Delta\phi_0=90°$ and linearly polarized light in the spin-zero representation. Interestingly, the α_C of elliptically polarized light with $\Delta\phi_0=90°$ changes 45° and $\Delta\phi_C$ changes 90° from the spin-zero representation to the spin representation, whereas the α_C of linearly polarized light changes from a variable angle to a fixed angle of 45°, the $\Delta\phi_C$ changes from a fixed angle of 0° to a variable angle of $2\alpha_0$.

In the spin representation, if the angle of incidence is equal to Brewster angle ($\theta_i=\theta_B$), and $\Delta\phi_C=180°$, then $\tilde{x}_R,\tilde{x}_L,\tilde{y}_R$ and $\tilde{y}_L$ expressions can be reduced to:

$$\tilde{x}_R=i\frac{1}{k^i}\left(-\frac{\partial r_P}{r_S\partial\theta_i}\bigg|_{\theta_i=\theta_B}\frac{\sin\left(\alpha_C+45°\right)}{\sin\left(\alpha_C-45°\right)}+\frac{\partial r_S}{r_S\partial\theta_i}\bigg|_{\theta_i=\theta_B}\right),\tag{9}$$

$$\tilde{x}_L=-i\frac{1}{k^i}\left(-\frac{\partial r_P}{r_S\partial\theta_i}\bigg|_{\theta_i=\theta_B}\frac{\sin\left(\alpha_C+45°\right)}{\sin\left(\alpha_C-45°\right)}-\frac{\partial r_S}{r_S\partial\theta_i}\bigg|_{\theta_i=\theta_B}\right),\tag{10}$$

$$\tilde{y}_R=\frac{1}{k^i}\frac{2\cos\alpha_C\cot\theta_B}{\cos\alpha_C-\sin\alpha_C},\tilde{y}_L=\frac{1}{k^i}\frac{2\sin\alpha_C\cot\theta_B}{\cos\alpha_C-\sin\alpha_C}.\tag{11}$$

From equations (9) to (11), the following relation can be easily obtained: $\Delta x^r_R=\Delta x^r_L=0$. According to equation (4), it is easy to obtain that the intensity of right-handed and left-handed light fields is equal ($I_R=I_L=\left|r_S(a^i_R-a^i_L)/2\right|^2$) in Brewster angle incidence. Therefore, when linearly polarized light in the spin representation impinges on the transparent medium interface, the spatial LSS of reflected light completely disappears, while the angular LSS at the α_C away from 45° shows an asymmetric LSS. The above result is because the $(\partial r_P/\partial\theta_i)\,|_{\theta_i=\theta_B}$ is on the same order of magnitude as the $(\partial r_S/\partial\theta_i)\,|_{\theta_i=\theta_B}$ [36], and cannot

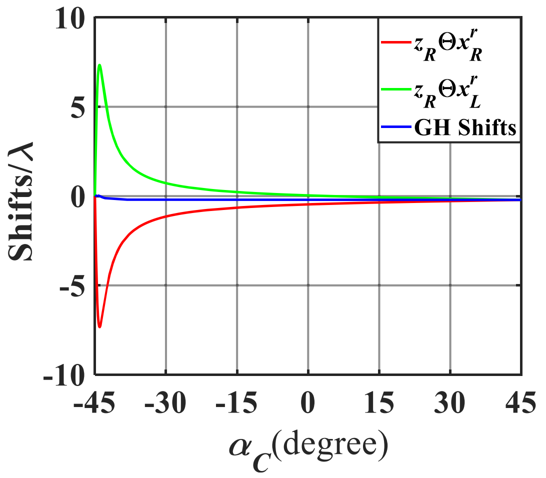

Figure 2. Angular LSS and GH shift in spin representation ($\Delta\phi_C = 180°$ at $\theta_i = \theta_B = 56.485°$) at air–glass interface ($n_1 = 1$, $n_2 = 1.5$).

satisfy that the former is much larger than the latter. For the α_C approaching 45°, the last term $(\frac{\partial r_S}{r_S \partial \theta_i})|_{\theta_i=\theta_B}$ in equations (9) and (10) can be ignored. Then, equation (6) can be simplified as:

$$\Theta x_L^r = -\Theta x_R^r. \tag{12}$$

This indicates that the angular LSS is a symmetric spin splitting. Therefore, the angular GH shift is zero. It is worth emphasizing that angular spin splitting of reflected light is usually an asymmetric spin splitting. Only when the elliptically polarized light with $\Delta\phi_0$ of 90° and α_0 of the near-zero in the spin-zero representation impinges on the air–glass interface at Brewster angle, angular LSS can be viewed as a symmetrical distribution about the origin of coordinates. The detailed behavior evolution of the spin optical effects is shown in figure 2.

In order to thoroughly reveal the generation mechanism of GH shift and LSS in spin representation, it is necessary to analyze the interference effect between the cross reflection field and the direct reflection field. The GH shift and LSS can be understood as three sources: one is the self-intensity of the spin component of the direct reflected light field, the other is the self-intensity of the cross-reflected light field, and the last is the interference effect of the direct reflected light field and the cross-reflected light field spin component. Specifically, for an incident linearly polarized light with an $\Delta\phi_C = 180°$ and $\alpha_C = -44.5°$, the self-intensity of the spin component of the directly reflected light field exhibits a tiny GH shift, and the GH shift is of the same magnitude and direction, that is, spin-independent splitting, as shown in figures 3(c) and (d). The GH shift of the self-intensity of the spin component of the cross-reflected light field almost disappears, as shown in figures 3(e) and (f). It is worth emphasizing that the interference effects of direct and cross-reflected light fields play a weakening role in the total intensity of the spin component of the reflected light field, as shown in figures 3(g) and (h). That is, the intensity of interference is a negative value, which is called negative interference. It is because of this interference phenomenon that the light intensity is very weak but large symmetric LSS, as shown in figures 3(a) and (b).

To more clearly show the physical meaning of the light field intensity represented in figure 3, the expression of the intensity of each sub-light field and the expression of the interference intensity between the direct reflected light field and the cross-reflected light field can be written as:

$$E_{RR}^{r}{}^{*}E_{RR}^{r} = \left|r_{RR}a_R^i\right|^2\left[1+|\tilde{x}_{RR}|^2\frac{k^{r2}}{z_R{}^2+z^{r2}}x^{r2}+|\tilde{y}_{RR}|^2\frac{k^{r2}}{z_R{}^2+z^{r2}}y^{r2}+2\,\mathrm{Re}\left(\tilde{x}_{RR}\xi_{x^r}+\tilde{y}_{RR}\xi_{y^r}\right)\right] \times\exp\left[-k^r\frac{x^{r2}+y^{r2}}{z_R{}^2+z^{i2}}z_R\right] \tag{13}$$

$$E_{LL}^{r}{}^{*}E_{LL}^{r} = \left|r_{LL}a_L^i\right|^2\left[1+|\tilde{x}_{LL}|^2\frac{k^{r2}}{z_R{}^2+z^{r2}}x^{r2}+|\tilde{y}_{LL}|^2\frac{k^{r2}}{z_R{}^2+z^{r2}}y^{r2}+2\,\mathrm{Re}\left(\tilde{x}_{LL}\xi_{x^r}+\tilde{y}_{LL}\xi_{y^r}\right)\right] \times\exp\left[-k^r\frac{x^{r2}+y^{r2}}{z_R{}^2+z^{i2}}z_R\right], \tag{14}$$

$$E_{RL}^{r}{}^{*}E_{RL}^{r} = \left|r_{RL}a_L^i\right|^2\left[1+|\tilde{x}_{RL}|^2\frac{k^{r2}}{z_R{}^2+z^{r2}}x^{r2}+2\,\mathrm{Re}\left(\tilde{x}_{RL}\xi_{x^r}\right)\right]\exp\left[-k^r\frac{x^{r2}+y^{r2}}{z_R{}^2+z^{i2}}z_R\right], \tag{15}$$

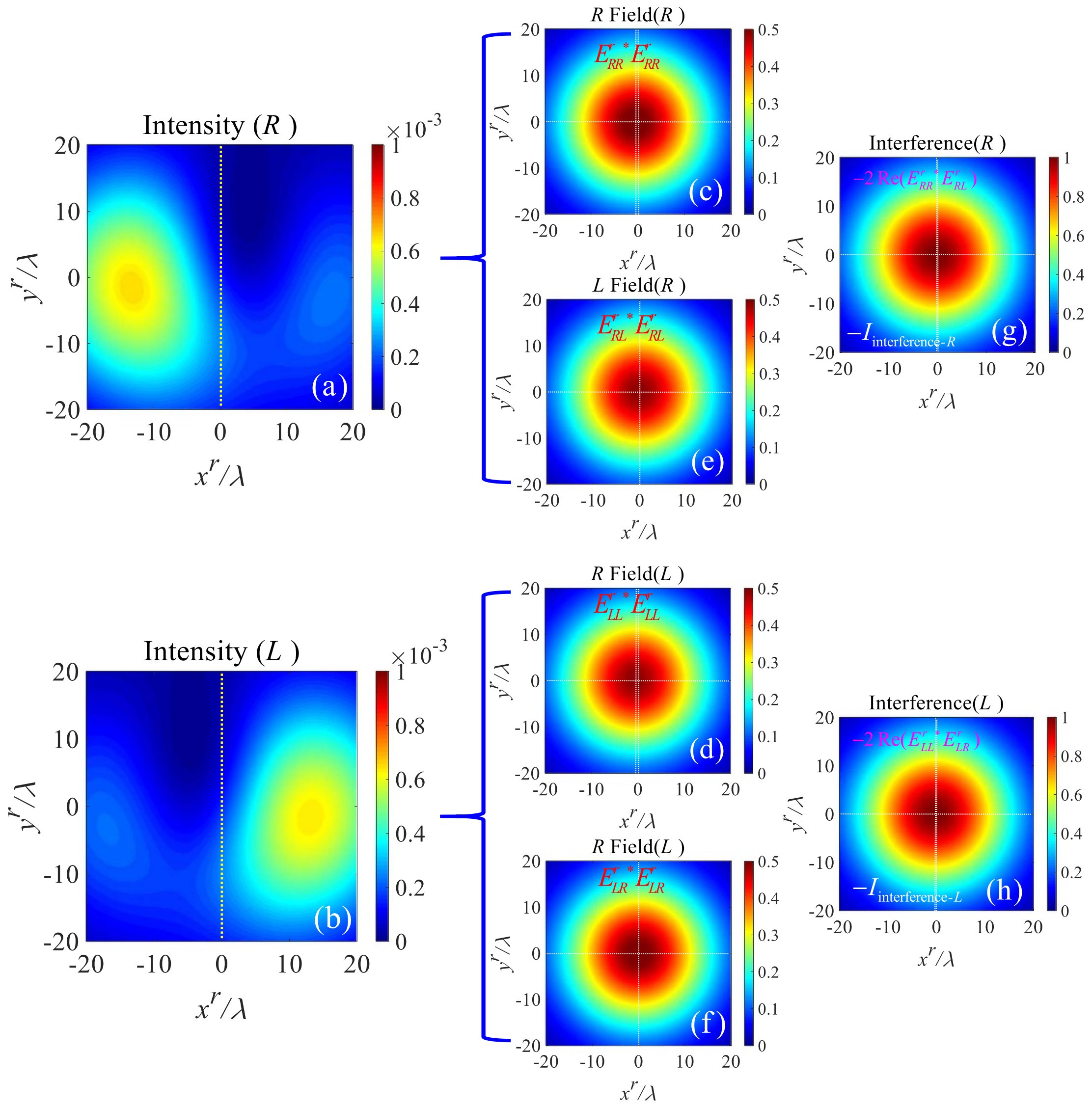

Figure 3. The intensity distribution of the spin components of the reflected light field from the air–glass interface at $\theta_i = \theta_B$ for the incident polarization state with $\alpha_C = -44.5°$ and $\Delta\phi_C = 180°$. (a) R and (b) L components for the total reflected light field; (c) R and (d) L components for the direction reflected light field; (e) R and (f) L components for the cross reflected light field; (g) R and (h) L components of the negative of the interference for the direct reflected light field and the cross reflected light field.

$$E^{r}_{LR}{}^{*}E^{r}_{LR} = \left|r_{LR}a^{i}_{R}\right|^{2}\left[1+|\tilde{x}_{LR}|^{2}\frac{k^{r2}}{z_R^{2}+z^{r2}}x^{r2}+2\,\mathrm{Re}\left(\tilde{x}_{LR}\xi_{x^r}\right)\right]\exp\left[-k^{r}\frac{x^{r2}+y^{r2}}{z_R^{2}+z^{i2}}z_R\right], \tag{16}$$

$$2\,\mathrm{Re}\left(E^{r}_{RL}{}^{*}E^{r}_{RL}\right) = 2\,\mathrm{Re}\left[r_{RR}{}^{*}r_{RL}a^{i}_{R}{}^{*}a^{i}_{L}\left(1+\tilde{x}^{*}_{RR}\frac{k^{r2}}{z_R^{2}+z^{r2}}x^{r2}\tilde{x}_{RL}+\tilde{x}_{RL}\xi_{x^r}+\tilde{x}^{*}_{RR}\xi_{x^r}{}^{*}+\tilde{y}^{*}_{RR}\xi_{y^r}{}^{*}+\tilde{y}^{*}_{RR}\xi_{y^r}{}^{*}\tilde{x}_{RL}\xi_{x^r}\right)\right]$$
$$\cdot\exp\left[-k^{r}\frac{x^{r2}+y^{r2}}{z_R^{2}+z^{i2}}z_R\right], \tag{17}$$

$$2\,\mathrm{Re}\left(E^{r}_{LL}{}^{*}E^{r}_{LL}\right) = 2\,\mathrm{Re}\left[r_{LL}{}^{*}r_{LR}a^{i}_{L}{}^{*}a^{i}_{R}\left(1+\tilde{x}^{*}_{LL}\frac{k^{r2}}{z_R^{2}+z^{r2}}x^{r2}\tilde{x}_{LR}+\tilde{x}_{LR}\xi_{x^r}+\tilde{x}^{*}_{LL}\xi_{x^r}{}^{*}+\tilde{y}^{*}_{LL}\xi_{y^r}{}^{*}+\tilde{y}^{*}_{LL}\xi_{y^r}{}^{*}\tilde{x}_{LR}\xi_{x^r}\right)\right]$$
$$\cdot\exp\left[-k^{r}\frac{x^{r2}+y^{r2}}{z_R^{2}+z^{i2}}z_R\right] \tag{18}$$

where the superscript * denotes the complex conjugation. Equations (13) and (14) represent the simulated intensity distribution in figures 3(c) and (d), respectively. The GH shift of the two spin component fields E^r_{RR} and E^r_{LL} of the direct reflected light field comes from the contribution of the $2\,\mathrm{Re}(\tilde{x}_{RR}\xi_{x^r})$ and $2\,\mathrm{Re}(\tilde{x}_{LL}\xi_{x^r})$ respectively, so the GH shift of the two spin component light fields is a spin-independent shift along the $-x$ direction. Equations (15) and (16) represent the simulated intensity distribution in figures 3(e) and (f), respectively. From $2\,\mathrm{Re}(\tilde{x}_{RL}\xi_{x^r}) = 2\,\mathrm{Re}(\tilde{x}_{LR}\xi_{x^r})$, it can also be concluded that the GH shift of the two spin components of the cross-reflected light field is a spin-independent shift along the $+x$ direction and its magnitude is almost negligible, as shown in figures 3(e) and (f). Interestingly, the interference intensity distributions described by equations (17) and (18) are shown in figures 3(g) and (h), respectively. According to $\mathrm{Re}(\tilde{x}_{RL}\xi_{x^r} + \tilde{x}^*_{RR}\xi_{x^r}{}^*) = \mathrm{Re}(\tilde{x}_{LR}\xi_{x^r} + \tilde{x}^*_{LL}\xi_{x^r}{}^*)$, it is easy to see that the GH shift of the two spin components of the interference field is still a spin-independent shift, and its magnitude is about half that of the GH shift of the directly reflected light field. It is worth emphasizing that the relative magnitude relationship between the GH shifts of the direct reflected light field, the cross-reflected light field, and the interference field is determined by $2\,\mathrm{Re}(\tilde{x}_{RR}\xi_{x^r}) + 2\,\mathrm{Re}(\tilde{x}_{RL}\xi_{x^r}) = 2\,\mathrm{Re}(\tilde{x}_{RL}\xi_{x^r} + \tilde{x}^*_{RR}\xi_{x^r}{}^*)$. However, this raises a new question: why does the spin component field of the total reflection field exhibit symmetric LSS when all three exhibit spin-independent GH shifts?

First, the longitudinal GH shifts of the right-handed and left-handed components of the total reflected light field can be expressed as:

$$\Delta x^r_R = \Delta x^r_{RR} P^r_{RR} + \Delta x^r_{RL} P^r_{RL} + \Delta x^r_{RI} P^r_{RI} \tag{19}$$

$$\Delta x^r_L = \Delta x^r_{LL} P^r_{LL} + \Delta x^r_{LR} P^r_{LR} + \Delta x^r_{LI} P^r_{LI}. \tag{20}$$

Here Δx^r_{RR}, Δx^r_{RL} and Δx^r_{RI} represent GH shifts shown in figures 3(c), (e) and (g), respectively, while Δx^r_{RR}, Δx^r_{RL} and Δx^r_{RI} represent GH shifts shown in figures 3(d), (f) and (h), respectively. P^r_{RR}, P^r_{RL} and P^r_{RI} (P^r_{LL}, P^r_{LR} and P^r_{LI}) represent the specific gravity factors of the GH shift terms Δx^r_{RR}, Δx^r_{RL} and Δx^r_{RL} (Δx^r_{LL}, Δx^r_{LR} and Δx^r_{LI}) respectively. Three points need to be highlighted about the characteristics of the specific gravity factor at Brewster angle of incidence: (1) the specific gravity factors of the GH shift caused by the interference term is negative because the interference intensity is negative, and the GH shift of the direct reflected light field and the GH shift of the cross-reflected light field respectively correspond to the specific gravity factor is obviously positive; (2) the relationship between specific gravity factors is as follows: $P^r_{RR} \neq P^r_{LL}$, $P^r_{RL} \neq P^r_{LR}$ and $P^r_{RI} = P^r_{LI}$; (3) the relationship between GH shift of direct reflected light field, cross-reflected light field and GH shift of interference field is as follows: $\Delta x^r_{RR} + \Delta x^r_{RL} = 2\Delta x^r_{RI}$ and $\Delta x^r_{LL} + \Delta x^r_{LR} = 2\Delta x^r_{LI}$. In addition, the superposition of the negative interference intensity, the self-intensity of the direct reflected light field and the self-intensity of the cross-reflected light field leads to the weak intensity of the total reflected light field, and further leads to the specific gravity factor of the order of 10 to the third power. Such a large specific gravity factor is the main reason why the GH displacement of the total reflected light field is enhanced. At Brewster angle incidence, the right-handed and left-handed components of the total reflected light field subtly appear as a huge LSS by adjusting the amplitude ratio angle α_C to change the specific gravity factors. The conditions under which this interesting phenomenon of symmetric LSS occurs are extremely strict.

In the spin representation, if the angle of incidence is equal to Brewster's angle ($\theta_i = \theta_B$), and $\alpha_C = 45°$, $\Delta\phi_C = 2\alpha_0$, then $\tilde{x}_R, \tilde{x}_L, \tilde{y}_R$ and $\tilde{y}_L$ expressions can be reduced to:

$$\tilde{x}_R = i\frac{1}{k^i}\left(\frac{\partial r_P}{r_S\partial\theta_i}\bigg|_{\theta_i=\theta_B}\frac{i\sin\Delta\phi_C}{1-\cos\Delta\phi_C} + \frac{\partial r_S}{r_S\partial\theta_i}\,|_{\theta_i=\theta_B}\right), \tag{21}$$

$$\tilde{x}_L = -i\frac{1}{k^i}\left(\frac{\partial r_P}{r_S\partial\theta_i}\bigg|_{\theta_i=\theta_B}\frac{i\sin\Delta\phi_C}{1-\cos\Delta\phi_C} - \frac{\partial r_S}{r_S\partial\theta_i}\,|_{\theta_i=\theta_B}\right), \tag{22}$$

$$\tilde{y}_R = \frac{1}{k^i}\frac{1-\cos\Delta\phi_C + i\sin\Delta\phi_C}{1+\cos\Delta\phi_C}\cot\theta_B, \tilde{y}_L = -\tilde{y}^*_R. \tag{23}$$

Obviously, the spatial LSS is an symmetric spin splitting, while the angular LSS completely disappears, i.e. $\Delta x^r_R = -\Delta x^r_L$, $\Theta x^r_R = \Theta x^r_L$. According to equations (7) and (8), it is not difficult to conclude that spatial GH shift is zero and angular GH shift occurs.

Finally, from the perspective of spin representation, we discuss the relationship between GH displacement and LSS in the case of total reflection from equations (7) and (8). As shown in figure 4(a), the magnitude of the two components of spatial LSS differs significantly except for the amplitude ratio angle $\alpha_C = 0°$ and $\alpha_C = \pm 45°$. We can clearly see that the variation characteristics of GH displacement are determined by the variation characteristics between the two components of the spatial LSS, and this result is completely consistent with the results obtained from the spin-zero representation reported previously [37]. More interestingly, angular LSS does not exhibit perfect symmetric spin splitting, although angular GH displacement always disappears, as shown in figure 4(b). It is also revealed that this interesting optical effect is mainly caused by the unequal ratio of the two spin components of the reflected light field (The variation trend of intensity ratio with amplitude ratio angle α_C is shown in figure 5). This is in stark contrast to the recently reported polarization-independent SHE for non-zero IF displacement by designing an interface structure with Fresnel coefficients satisfying $r_P = r_S$ ($r_{PS} = r_{SP} = 0$) [38]. In particular, in the spin-zero representation, the incident circularly polarized light retains only one spin component light field after

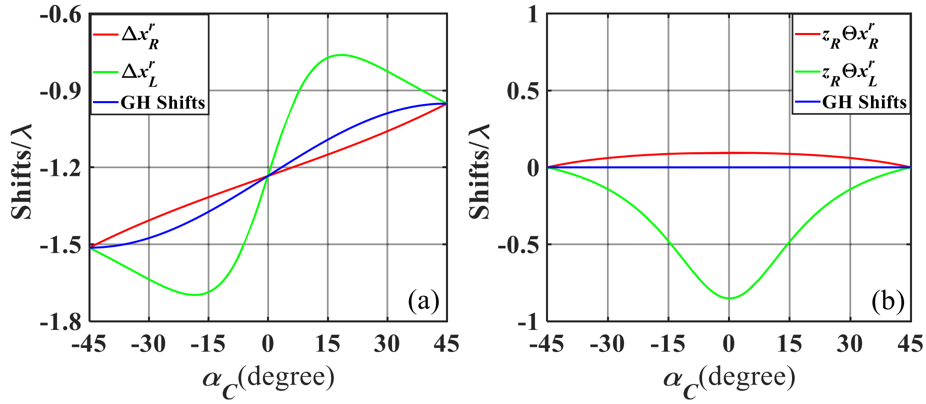

Figure 4. The spatial and angular LSS and GH in the spin representation when the light impinges on the glass–air interface for total reflection ($\Delta\phi_C = 180°$, $\theta_i^C = 45°$, $n_1 = 1.5$, $n_2 = 1.0$).

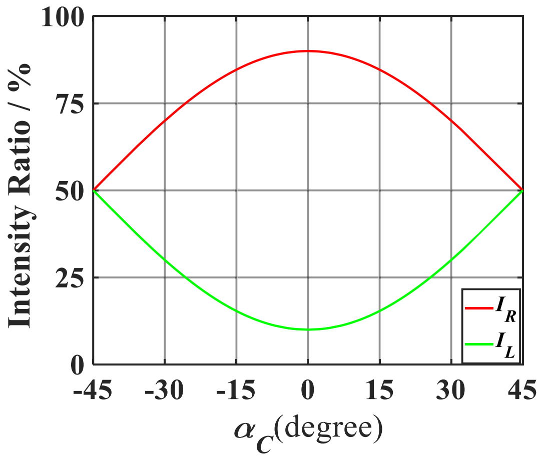

Figure 5. The relationship between the intensity ratio of the right-handed component and the left-handed component of the reflected light field under the condition of total reflection ($\Delta\phi_C = 180°$, $\theta_i^C = 45°$, $n_1 = 1.5$, $n_2 = 1.0$).

reflection, which means that the SHE of circularly polarized light completely disappears at this time. It is worth emphasizing that the occurrence of the SHE of light predicted by Bliokh is closely related to IF displacement being zero [4]. From this perspective, it is obvious that the so-called polarization-independent SHE cannot occur. Therefore, the relationship between spatial IF displacement and transverse spin splitting derived from our theory also play an important role in analyzing the difference and connection between SHE and transverse symmetric spin splitting of light.

4. Conclusion

In summary, we have described the GH shift and LSS of arbitrarily polarized light in spin representation by polarization representation transformation. We have also revealed the mechanism of GH shift from the interference effect of direct and cross-reflected light fields, and established the relationship between GH shift and LSS in spin representation. Remarkably, we have strictly proved that angular symmetric spin splitting of reflected light is generally impossible to occur when linearly polarized light in the spin representation strikes the air–glass interface at Brewster angle. Finally, from the perspective of spin representation, we also reveal the physical mechanism of the optical effect that angular GH displacement always disappears but angular LSS does not exhibit perfect symmetric spin splitting in the case of total reflection.

Data availability statement

The data cannot be made publicly available upon publication because no suitable repository exists for hosting data in this field of study. The data that support the findings of this study are available upon reasonable request from the authors.

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Appendix. Transformation matrix between the incident and reflected light fields in the spin representation

According to the electric field expression equation (2) of the incident beam in the spin representation, the spin representation of the electric field of the incident light beam in the local coordinate system can be expressed as:

$$\left|\tilde{\boldsymbol{E}}_S^{i'}\right\rangle = \hat{V}\hat{U}_{\theta_i\perp}\left(\theta_i,\boldsymbol{k}_i\right)\hat{V}^{\dagger}\left|\tilde{\boldsymbol{E}}_S^{i}\right\rangle, \tag{A1}$$

with $\hat{V}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}$, $\hat{U}_{\theta_i\perp}(\theta_i,\boldsymbol{k}_i)=\begin{pmatrix}1 & \frac{k_y^i}{k^i}\cot\theta_i\\ -\frac{k_y^i}{k^i}\cot\theta_i & 1\end{pmatrix}$.
where $\hat{V}$ is the transformation matrix of the incident light field from spin representation to spin-zero representation, $\hat{U}_{\theta_i\perp}(\theta_i,\boldsymbol{k}_i)$ is the transformation matrix of the incident light field from the incident laboratory coordinate system to the incident local coordinate system. According to the Fresnel equation in the spin representation, the electric field of the reflected light in the local coordinate system can be written as:

$$\left|\tilde{\boldsymbol{E}}_S^{r'}\right\rangle = \hat{S}^r\hat{F}_S^r\left|\tilde{\boldsymbol{E}}_S^{i'}\right\rangle, \tag{A2}$$

with $\hat{F}_S^r=\begin{pmatrix}r_{RR} & r_{RL}\\ r_{LR} & r_{LL}\end{pmatrix}$, $r_{RR}=r_{LL}=\frac{r_P+r_S}{2}$; $r_{LR}=r_{RL}=\frac{r_P-r_S}{2}$.

The operator $\hat{S}^r$ in equation (A2) can be regarded as a coordinate transformation operator from (k_x^i,k_y^i) to (k_x^r,k_y^r), i.e. $\hat{S}^r\left|\tilde{E}^r(k_x^i,k_y^i)\right\rangle=\left|\tilde{E}^r(-k_x^r,k_y^r)\right\rangle$. It is worth emphasizing that the appearance of the Fresnel non-zero matrix elements ($r_{RL}=r_{LR}\neq 0$) in the spin representation clearly indicates that the eigenbasis of the reflected light fields is the linearly polarized basis rather than the circularly polarized basis. r_{RR}, r_{LL} and r_{LR}, r_{RL} are similar to the direct Fresnel reflection coefficients and cross Fresnel reflection coefficients of anisotropic interface reflection. Interestingly, the cross-Fresnel reflection coefficient is due to the polarization representation transformation. Next, we give a simple derivation of the Fresnel equation for the reflected light field in spin representation.

Fresnel equation for the reflected light field in a spin-zero representation can be expressed as:

$$\left|\tilde{\boldsymbol{E}}^{r'}\right\rangle = \hat{F}^r\left|\tilde{\boldsymbol{E}}^{i'}\right\rangle, \tag{A3}$$

with $\hat{F}^r=\begin{pmatrix}r_P & 0\\ 0 & r_S\end{pmatrix}$.

Multiply the right side of the Fresnel reflection matrix $\hat{F}^r$ in equation (A3) by the identity matrix $\hat{I}=\hat{V}^{\dagger}\hat{V}$, the following relation is obtained:

$$\left|\tilde{\boldsymbol{E}}^{r'}\right\rangle = \hat{F}^r\hat{V}^{\dagger}\hat{V}\left|\tilde{\boldsymbol{E}}^{i'}\right\rangle. \tag{A4}$$

Then multiply both sides of the equation (A4) left by the transformation operator $\hat{V}$ of the spin representation to obtain the important expression that follows

$$\hat{V}\left|\tilde{\boldsymbol{E}}^{r'}\right\rangle = \hat{V}\hat{F}^r\hat{V}^{\dagger}\hat{V}\left|\tilde{\boldsymbol{E}}^{i'}\right\rangle. \tag{A5}$$

By using the relation: $\hat{V}\left|\tilde{\boldsymbol{E}}^{r'}\right\rangle=\left|\tilde{\boldsymbol{E}}_S^{r'}\right\rangle$, $\hat{V}\left|\tilde{\boldsymbol{E}}^{i'}\right\rangle=\left|\tilde{\boldsymbol{E}}_S^{i'}\right\rangle$, Thus, the Fresnel equation of the reflected light beam in spin representation can be written as:

$$\left|\tilde{\boldsymbol{E}}_S^{r'}\right\rangle = \hat{F}_S^r\left|\tilde{\boldsymbol{E}}_S^{i'}\right\rangle, \tag{A6}$$

with $\hat{F}_S^r=\hat{V}\hat{F}^r\hat{V}^{\dagger}$.

On the basis of equation (A6) and considering the mirror symmetry of the coordinate transformation of reflection process, equation (A2) is obtained. For the convenience of calculating the GH displacement next, the first-order Taylor expansion of the Fresnel coefficient in the spin representation is performed at $(k_x^i = 0, k_y^i = 0)$:

$$\tilde{r}_{ij}\left(\theta_i, \boldsymbol{k}^i\right) = \tilde{r}_{ij}\Big|_{k_x^i=0;k_y^i=0} + \partial_{k_x^i}\tilde{r}_{ij}\Big|_{k_x^i=0;k_y^i=0} k_x^i, ij = RR, RL, LR, LL. \tag{A7}$$

Therefore, the electric field of reflected light in the local laboratory coordinate system can be obtained as:

$$\left|\tilde{\boldsymbol{E}}_S^r\right\rangle = \hat{V}\hat{U}^\dagger_{\theta_r\perp}\left(\theta_r, \boldsymbol{k}_r\right)\hat{V}^\dagger\left|\tilde{\boldsymbol{E}}_S^{r'}\right\rangle. \tag{A8}$$

So far, we have obtained the relationship between the electric field of incident light and the electric field of reflected light in the spin representation, that is:

$$\left|\tilde{\boldsymbol{E}}_S^r\right\rangle = \hat{M}_S^r\left|\tilde{\boldsymbol{E}}_S^i\right\rangle,$$
$$\hat{M}_S^{i\mapsto r} = \hat{V}\hat{U}^\dagger_{\theta_r\perp}\left(\theta_r, \boldsymbol{k}_r\right)\hat{V}^\dagger\hat{S}^r\hat{F}_S^r\hat{V}\hat{U}_{\theta_i\perp}\left(\theta_i, \boldsymbol{k}_i\right)\hat{V}^\dagger. \tag{A9}$$

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